Table of Contents

Volume 11, Number 4

2006

Military Applications Society

Institute for Operations Research and the Management Sciences

The Military Applications Society (MAS) is a democratically constituted professional society of open membership dedicated to the free and open pursuit of the science, engineering and art of military operations. It is the first society of the Institute for Operations Research and the Management Sciences (INFORMS). MAS advances research in military operations, fosters higher standards of practice of military operations research, promotes the exchange of information among developers and users of military operations research; and encourages the study of military operations research.

The society sponsors two open meetings annually, publishes a book series "Topics in Operations Research," and, among other activities, jointly publishes *PHALANX* and *MILITARY OPERATIONS RESEARCH* with the Military Operations Research Society.

President
Dr. Edward A. Pohl
Department of Industrial Engineering
University of Arkansas
Fayetteville, AR 72701
Phone: 479-575-6042
epohl@engr.uark.edu

Vice President/President Elect
Dr. Patrick J. Driscoll
Department of Systems Engineering
United States Military Academy
Mahan Hall, Room 301
West Point, NY 10996-1779
Phone: 845-938-6587
patrick.driscoll@usma.edu

Secretary/Treasurer
LTC Brian K. Sperling
Department of Systems Engineering
United States Military Academy
Mahan Hall, Room 301
West Point, NY 10996-1779
Phone: 845-938-4399
brian.sperling@usma.edu

Awards Committee Chair and Former President
Dr. Keith N. Womer
Hearin Center for Enterprise Science
University of Mississippi
662-915 5820
kwomer@bus.olemiss.edu

Council Member
Dr. Alan W. Johnson
Air Force Institute of Technology
Dept of Operational Sciences
Phone: 937-255-3636 x 4703
alan.johnson@afit.edu

Dr. Russ R. Vane III
General Dynamics
Phone: 703-450-0137
russ.vane@gd-ais.com

Council Member
LTC Timothy E. Trainor, USA, Ph.D.
Department of Systems Engineering
United States Military Academy
Phone: 845-938-5534
tim.trainor@usma.edu

Dr. Roger C. Burk
Dept. of Systems Engineering
United States Military Academy
Phone: 845-938-4754
roger.burk@usma.edu

Former President
Dr. Richard F. Deckro
Air Force Institute of Technology
richard.deckro@afit.af.mil

Philipp A. Djang
US Army Research Labs
djang@arl.army.mil

Former President
Dr. John P. Ballenger
Raytheon, Inc.
jp_ballenger@raytheon.com

Dr. Bruce W. Fowler
US Army Aviation Missile Command
fowler-bw@redstone.army.mil

Former President
Dr. Tom R. Gulledge
George Mason University
gulledge@gmu.edu

Dr. Steve J. Balut
Institute for Defense Analysis
sbalut@ida.org

Executive Summaries

PROBABILITY MODELING OF AUTONOMOUS UNMANNED COMBAT AERIAL VEHICLES (UCAVs)

by Moshe Kress, Arne Baggesen and Eylam Gofer

Advances in sensors and command, control, communications, computers, intelligence, surveillance and reconnaissance (C4ISR) technologies, coupled with operational needs, like the war against terror, have led in recent years to the development of a new class of weapon systems called *Unmanned Combat Aerial Vehicles*, or in short—UCAVs. Autonomous UCAVs combine a unique set of capabilities in one platform; they have an *eye* that senses the area and gathers target information, a *brain* that processes this information, *wings* that move the UCAV around and keep it aloft and a *fist*, in a form of a warhead. This paper addresses several design and operational issues related to the employment of UCAVs. In particular, we study tradeoffs among properties related to the eye, brain, wing and fist such as detection, situational awareness, memory, coordination, vulnerability and lethality.

DISTORTED RISK MEASURES WITH APPLICATION TO MILITARY CAPABILITY SHORTFALLS

by Edwin J. Offut, Jeffrey P. Kharoufeh and Richard F. Deckro

In today's environment of transformation, budget restrictions, asymmetric conflict, and evolving technologies, it is essential that the risks associated with military capability shortfalls are correctly modeled and evaluated, especially when low-likelihood events result in potentially catastrophic losses. This study focuses on selecting an appropriate distortion function and associated parameters to account for rare but catastrophic events that may result from shortfalls in military capabilities. Using a notional example, we illustrate how our approach might be applied within the context of resource allocation.

This work was selected as Best Working Group Paper in WG 21, Readiness, at the *73rd MORS Symposium.*

ESTIMATING TOTAL PROGRAM COST OF A LONG-TERM, HIGH-TECHNOLOGY, HIGH-RISK PROJECT WITH TASK DURATIONS AND COSTS THAT MAY INCREASE OVER TIME

by Gerald G. Brown, Roger T. Grose, and Robert A. Koyak

The U.S. Army's Future Combat Systems (FCS) exemplifies the challenges of scheduling large-scale military acquisitions. The Cost Analysis Improvement Group (CAIG) in the Program Analysis and Evaluation (PA&E) branch of OSD wanted to compare three different schedule plans for FCS. Brown, Grose, and Koyak develop an innovative application of integer programming, combined with simulation, which brings greater realism into analyzing alternate scheduling plans. Using FCS to demonstrate their approach, the authors show how useful comparisons can be made taking into account uncertainty in task durations and budget constraints over the planning cycle of the project.

Call for Papers
Military Operations Research
Special Issue: Value-Focused Thinking Applied to the Challenges of the Long War

Introduction

Stakeholder values pervade decision making in all fields and are of particular importance in complex military decision making in uncertain future environments. Most decision making processes have some method to evaluate the consequences of the decision alternatives. Value-Focused Thinking (VFT) (Keeney, 1992) is a philosophy that starts by identifying values and then uses these values to *evaluate* and *improve* alternatives. VFT uses the mathematics of Multiple Objective Decision Analysis (MODA) (Keeney & Raiffa, 1976; Kirkwood, 1997) to quantify the values and evaluate the alternatives. This technique is appropriate when we have conflicting objectives, complex alternatives, and major uncertainties. As the Department of Defense has come to increasingly require understandable and traceable analysis, VFT using MODA analysis has gained increasing acceptance.

This special issue of *Military Operations Research* recognizes the increasing potential of VFT/MODA for military decision making. While VFT/MODA has been used successfully in several applications, not all applications have been successful. We seek to highlight the successes and identify the political, technical, organizational, and emotional challenges that have limited VFT/MODA applications.

Topics of interest to the special issue include, but are not limited to, the following:

- Homeland Defense;
- Capabilities-Based Planning;
- Effects-Based Operations;
- Systems Engineering;
- Budgetary Planning and Programming;
- Analysis of Alternatives;
- Disaster Response;
- Theoretical extensions of VFT/MODA for military decision making;
- The use of VFT/MODA to extend and improve other forms of military decision making;
- Use of VFT/MODA with modeling and simulation; and
- Pitfalls to avoid in using VFT/MODA

Paper Submission Due Date: 31 January 2007

You are invited to send a PDF file or three hard copies of your manuscript for publication consideration to the Special Issue Editors, *Military Operations Research*

Drs. Gregory Parnell, FS and Raymond Hill, *MOR* Special Issue Editors, Military Operations Research Society, 1703 N. Beauregard Street, Suite 450, Alexandria, VA 22311,
gregory.parnell@usma.edu or ray.hill@wright.edu

Ensure your cover letter indicates consideration is for this special issue. Once the special issue is complete, authors whose papers are not in the special issue may be invited to submit their papers for regular review by the journal.

Final paper submissions: If accepted, each final submission must be prepared based on the *MOR* Journal Editorial Policy: http://www.mors.org/publications/mor/mor_ed-policy.pdf

For questions please contact the *MOR* Special Issue Editors: Professor Gregory S. Parnell, FS, United States Military Academy, gregory.parnell@usma.edu OR Professor Raymond R. Hill, Wright State University, Ray.Hill@wright.edu.

Probability Modeling of Autonomous Unmanned Combat Aerial Vehicles (UCAVs)

Dr. Moshe Kress

Naval Postgraduate School
mkress@nps.edu

Dipl.-Ing. Arne Baggesen

Department for Sensors and Weapons
German Naval Office

Eylam Gofer

CEMA—Center for Military Analyses, Israel
eyalmg@rafael.co.il

APPLICATION AREA:
Unmanned Systems, Strike Warfare
OR METHODOLOGY:
Markov Processes

ABSTRACT

Unmanned Combat Aerial Vehicles (UCAVs) are advanced weapon systems that can loiter autonomously in a pack over a target area, detect and acquire the targets, and then engage them. Modeling these capabilities in a specific hostile operational setting is necessary for addressing weapons' design and operational issues. In this paper we develop several analytic probability models, which range from a simple regenerative formula to a large-scale continuous-time Markov chain, with the objective to address the aforementioned issues. While these models capture key individual aspects of the weapon such as detection, recognition, memory and survivability, special attention is given to pack related aspects such as simultaneous targeting, multiple kills due to imperfect battle damage assessment, and the effect of attack coordination. From implementing the models we gain some insights on design and operational considerations regarding the employment of a pack of UCAVs in a strike scenario.

INTRODUCTION

Advances in sensors and command, control, communications, computers, intelligence, surveillance and reconnaissance (C4ISR) technologies, coupled with operational needs, like the war against terror, have led in recent years to the development of a new class of weapon systems called *Unmanned Combat Aerial Vehicles*, or in short-UCAVs. A UCAV is a self-propelled aerial vehicle that typically loiters over the target area, seeking targets for engagement. UCAVs combine a unique set of capabilities in one platform; they have an *eye* that senses the area and gathers target information, a *brain* that processes this information, *wings* that move the UCAV around and keep it aloft and a *fist*, in a form of a warhead. There are two major types of UCAVs: disposable and retrievable. Disposable UCAVs are essentially precision guided munitions (PGM), like guided missiles, where the warhead is an integral part of the platform. Thus, a UCAV of this type can engage at most one target. Examples of disposable UCAVs are the Israeli Harpy (Jane's, 2000a), the German Taifun (Jane's, 2000b) and the US (Lockheed Martin) LOCAAS (Jane's, 2002). Retrievable UCAVs are larger vehicles that carry one or more munitions, which are launched from the vehicle towards the targets in a controlled trajectory. Once the weapons are expended, the UCAV returns to its base for refit and reload. An example of a retrievable UCAV is the US Air Force Predator that can carry a Hellfire laser-guided missile (Airforce Technology 2005).

In this paper we focus on *autonomous* UCAVs, which are designed to operate as a pack of vehicles that autonomously search, detect, acquire and attack targets. Similar operational concepts are imbedded in the Autonomous Wide Area Search Munition (AWASM), which is developed by Lockheed Martin for the US Air Force (Lockheed Martin, 2004). While much attention is given to the engineering and technological aspects of UCAV developments, there are very few studies on operational concepts for these weapon systems and their expected effectiveness and efficiency. The wide range of design and operational factors and capabilities of such autonomously acting and interacting weapons will most likely lead to a wide range of engagement performance in various scenarios. The problems are to select proper measures of effectiveness (MOEs) for the engagement performance, map the functional relations between the parameters and the MOEs, and obtain insights regarding the design of the UCAVs and their tactical employment.

While target detection and recognition capabilities, and weapon's accuracy and lethality determine the effectiveness of a single vehicle, two phenomena may affect the performance of the UCAVs as a pack: *multiple acquisitions* and *multiple kills.* Multiple acquisitions occur when two or more UCAVs acquire, and are about to engage, the same target. This situation, which may lead to redundancy and waste of attack resources, is due to lack of targeting coordination among the UCAVs. Absent multiple acquisitions, multiple kills occur when a UCAV engages a target that has already been killed by another UCAV. This situation is due to imperfect battle damage assessment (BDA).

The issue of coordination and cooperative control for target acquisition is addressed in several studies. Jacques (2002) presents a simple probability model for examining some operational aspects of employing a pack of AWASM. Other studies (e.g., Chandler et al (2002), Gillen and Jacques (2002) and Richards et al (2002)) utilize simulations for evaluating possible information sharing schemes, and develop optimization (mixed-integer programming) models that produce task assignment rules for target observation and classification and trajectory designs. Jeffcoat (2004) applies Markov-chain analysis to study the effect of cueing in the case of two cooperative searchers. The effect of BDA is analyzed in the context of Shoot-Look-Shoot models. Aviv and Kress (1997) utilize Markov and dynamic programming models to evaluate several shooting tactics when damage information is only partial. Manor and Kress (1997) prove the optimality of a certain shooting tactics under conditions of incomplete information. An optimal assignment of weapons and BDA sensors is presented in Yost and Washburn (2000), and a general review of probability models for evaluating Shoot-Look-Shoot models in the presence of partial damage information is given in Glazebrook and Washburn (2004).

In this paper we develop analytic probability models for analyzing some design and operational aspects relating to autonomous UCAVs. The models range from a simple regenerative formula to a large scale continuous-time Markov chain. In addition to considering individual UCAV properties—detection, recognition, memory, kill-effectiveness and vulnerability—the models explicitly incorporate also the effect of multiple acquisitions and multiple kills. Unlike simulations, a single run of each of these models produces *exact* probability distributions and values for the MOEs, and by applying these models to a set of design and operational parameters some insights—not all intuitive—are gained. The rest of the paper is organized as follows. In the next section we describe the basic operational setting of the situation we model, and in the following section we introduce notation and discuss the basic assumptions. In "Does Memory Matter?" we address the issue of UCAV memory and answer the question "is it an important feature?" Some transient properties of the engagement process in the case of no situational awareness are presented in the "Multiple UCAVs, No Situational Awareness" section. Following this, we study the complete problem where both multiple acquisition and situational awareness are considered. We formulate the continuous-time Markov model and present the results of the analysis, along with some design and operational insights. Finally, a summary and concluding remarks are presented.

THE BASIC SITUATION

A pack of single-weapon autonomous unmanned combat aerial vehicles (UCAV) is launched on a mission to attack a set of homogeneous targets located on the ground or at sea in a specific target area. Each UCAV loiters independently over the target area searching for *valuable targets.* The definition of a valuable target depends on the scenario and mission e.g., armored vehicles in tactical ground combat scenarios, air-defense missile launchers and radar sites in suppression of air defense (SEAD) missions, and command posts in operational-level missions. All other targets are *non-valuable.* A killed valuable target becomes non-valuable.

During its mission, a UCAV can be in one of three possible situations: *search*, *attack* or *removed.* A UCAV is said to be *searching* if it is still loitering and it has not acquired a target for engagement yet. Once a UCAV detects a target it locks on the target and attempts to identify if it is a valuable or non-valuable target. If the UCAV classifies the target (correctly or incorrectly) as non-valuable, the target is *rejected* (not acquired), the UCAV disengages and moves on with its search. If the UCAV classifies a target as valuable, it acquires the target and attacks it. The randomly distributed *inter-detection* time of a UCAV in a search stage is defined as the time between two consecutive detections of targets. This time comprises the loitering time from the last rejection to a new detection, and the identification time between the moment of lock-on and the moment the UCAV identifies the target and decides to attack (in case of acquisition) or disengage (in case of rejection). The total search time of a UCAV is the sum of its inter-detection times. We assume that the inter-detection times are not dependent on the classification result. The randomly distributed *attack time* is measured from the moment the target is classified as valuable to the moment the weapon hits the ground (or the surface). Figure 1 describes the aforementioned mission time parameters.

Once a UCAV enters an attack stage, it is committed to attack the acquired target and therefore cannot go back to the search stage, even if during the time of the attack another UCAV hits the target and kills it. Thus, if several UCAVs acquire the same target, at most one of them can be effective. We consider a UCAV that is either disposable or carries a single missile, therefore after an attack the UCAV is removed from further consideration in the current mission. A UCAV may *fail* during the search or attack stages if it is intercepted by enemy's air defense or it crashes due to technical failure or accident. We assume imperfect sensitivity and specificity; therefore identification may be subject to error. A valuable target may be identified, due to imperfect sensitivity, as non-valuable and therefore passed over by the UCAV, and a non-valuable target may be identified, due to imperfect specificity, as valuable and therefore attacked by the UCAV. We assume that the nominal loitering time (e.g., due to fuel consumption) is long compared to the minimum between the time it takes a UCAV to acquire and attack a target and the time until it (possibly) fails. In other words, a UCAV never runs out of fuel before its mission is over.

Given this combat situation, we wish to measure the effectiveness of the UCAVs, perform sensitivity analysis, and determine tradeoffs among design and operational parameters.

NOTATION AND BASIC ASSUMPTIONS

The probabilities of correctly identifying a valuable target and correctly identifying a non-valuable target are q_1 and q_2, respectively. That is, q_1 represents the *sensitivity* of the UCAV's sensor and data processing unit, and q_2 their *specificity*. The identification attempts are independent. The sensitivity and specificity of a UCAV determine its *BDA capabilities.* BDA (battle damage assessment) refers to the ability of a shooter to distinguish between a live valuable target and a killed one (which becomes non-valuable). For simplicity we assume that the specificity of the UCAV with respect to initially non-valuable targets is the same as for killed valuable targets. The models can be easily generalized to account for target dependent specificity. An acquired target is successfully hit and killed with probability p. To simplify the model, and without loss of generality, we assume that the probability of a kill given a hit is 1. We assume that the inter-detection and the attack times are exponentially distributed random variables with parameters λ and μ, respectively. While the former is a reasonable assumption based on the independent and memory-less nature of the search process (see Section 4 below), the latter is an approximation, which is similar to the exponential inter-firing assumption in stochastic duel or stochastic Lanchester models (e.g., Kress (1991) and Kress and Talmor (1999)). The failure rate of UCAVs

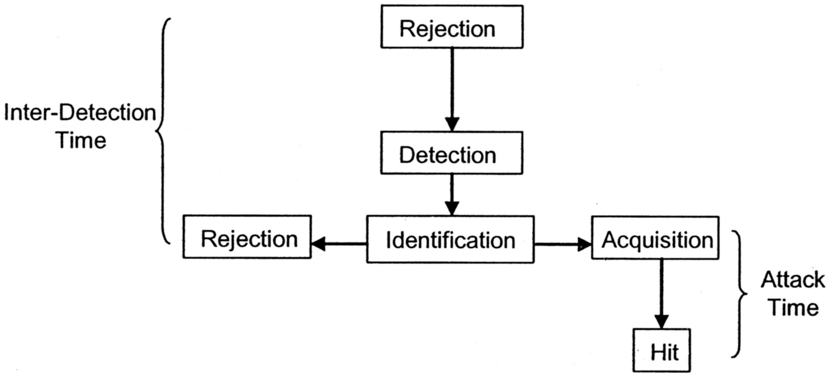


Figure 1. UCAV Mission Timeline.

is assumed to be constant and therefore the time until a UCAV fails is exponentially distributed random variable with parameter θ. The launched pack comprises N UCAVs. The total number of targets—valuable and non-valuable—in the target zone at the beginning of the operation is T, out of which L targets are valuable and T-L targets are non-valuable.

DOES MEMORY MATTER?

Consider a single UCAV, which detects a target and decides, correctly or incorrectly, to reject it. This event may or may not register in the UCAV's memory. If the UCAV remembers the rejected targets, then it would not consider any of them for future acquisition and therefore, after a finite number of detections, the pool of potential targets for engagement may be depleted. Absent memory, and since the detections are independent, it is possible that the UCAV will acquire a previously rejected target. The question is, can memory enhance (or reduce) the probability that the search process terminates with a killed valuable target?

First we assume no memory (NM). That is, the UCAV may detect and examine the same target more than once. The probability $P_{NM}(T,L)$ that a UCAV acquires and kills a valuable target, given there are a total of T targets and L valuable targets in the target area, satisfies the following regenerative equation:

$$P_{NM}(T, L) = \underbrace{\frac{\lambda}{\lambda + \theta}}_{\substack{\text{Probability of}\\ \text{successful}\\ \text{detection}}} \cdot \left[\underbrace{\frac{\mu}{\mu + \theta} \cdot \frac{L}{T} \cdot q_1 \cdot p}_{\substack{\text{Probability of}\\ \text{successful attack}}} + \underbrace{\left(\frac{L}{T} \cdot (1 - q_1) + \frac{T - L}{T} \cdot q_2 \right)}_{\text{Probability the target is rejected}} \cdot P_{NM}(T, L) \right]. \tag{1}$$

The solution of (1) is:

$$P_{NM}(T, L) = P(\alpha) = \frac{\mu}{\mu + \theta} \cdot \frac{\alpha \cdot \lambda \cdot q_1 \cdot p}{\lambda + \theta \cdot (1 + \alpha) + \alpha \cdot \lambda \cdot q_1 - \lambda \cdot q_2} \tag{2}$$

where $\alpha = L/(T - L)$. That is, the acquisition probability depends on the ratio between the numbers of valuable and non-valuable targets and not on their absolute numbers. Also it depends on the *endurance ratios* λ/θ and μ/θ, and not on the absolute values of the detection, attack and failure intensities.

Suppose now an ideal situation where the UCAV has perfect memory and situational awareness and therefore it would always detect and examine a new target. In that situation it is possible that the search process will terminate with no acquisition. In that case, we assume that the UCAV instantaneously selects any of the T targets at random and attacks it. This termination condition is appropriate in particular in time-critical missions. Since previously detected targets are automatically discarded from the search process, the rate at which new targets are detected decreases as the number of detected targets increases. Specifically, if the nominal detection rate at the beginning of the operation is λ then after k detected (and rejected) targets, the rate at which new targets are detected is $\lambda(1 - k/T)$. The probability of killing a valuable target $P_M(T,L)$ is

$$P_M(T, L) = q_1 \cdot p \cdot \underbrace{\frac{\mu}{\mu+\theta} \cdot \sum_{i=0}^{L-1} \sum_{j=0}^{T-L} \frac{\binom{L}{i}\binom{T-L}{j}}{\binom{T}{i+j}} \cdot \frac{L-i}{T-(i+j)} \cdot \prod_{k=0}^{i+j} \left(\frac{\lambda(1-k/T)}{\lambda(1-k/T)+\theta} \right)}_{\text{Probability that a target was successfully attacked in one of the } T \text{ detections}} \cdot (1-q_1)^i \cdot q_2^j \tag{3}$$

$$+ \underbrace{\frac{\mu}{\mu+\theta} \cdot \frac{L}{T} \cdot p \cdot \prod_{k=0}^{T-1} \left(\frac{\lambda(1-k/T)}{\lambda(1-k/T)+\theta} \right) \cdot (1-q_1)^L \cdot q_2^{T-L}}_{\substack{\text{Probability that all } T \text{ detections resulted in rejection and therefore} \\ \text{the target for attack is chosen randomly}}}$$

.

Suppose that the target area contains a total of 16 targets. We consider three target postures in which the proportion of valuable targets $\frac{L}{T}$ are 1/4 (e.g., a section of armored vehicles), 1/2 (e.g., two sections) and 3/4 (e.g., a company). For each one of the target postures we consider two *endurance ratios* $\frac{\lambda}{\theta}$; 20 (high survivability rate), and 3 (low survivability rate). We assume also two attack situations: slow execution where $\mu = \lambda$ and fast execution, where $\mu = 10\lambda$. In all the scenarios we assume that the hit probability given acquisition $p = 1$, which means that P is in fact the acquisition probability. Note that p is a multiplicative factor that does not affect the relative effectiveness of the no memory and full memory cases. For each one of the twelve scenarios we evaluate the kill (acquisition) probability P for various values of sensitivity probability q_1 and specificity probability q_2. Tables A1–A3 in the Appendix detail the results of the analysis for three target postures: $\frac{L}{T} = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$, respectively. Figures 2–4 present the comparison between the no-memory (black lines) and full-memory (grey lines) cases for the three target postures $\left(\frac{L}{T} = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right)$ with $\frac{\lambda}{\theta} = 20$ and $\mu = 10\lambda$.

Clearly, P is monotonic increasing in both q_1 and q_2; better sensitivity and specificity results in higher acquisition probability. While for some (relatively small) values of q_1 and q_2 the no-memory system outperforms the full-memory system, and for other (relatively large) values the opposite is true, the differences between the two cases are negligible. This conclusion is robust with respect to the detection, attack and failure rates (see Appendix). For

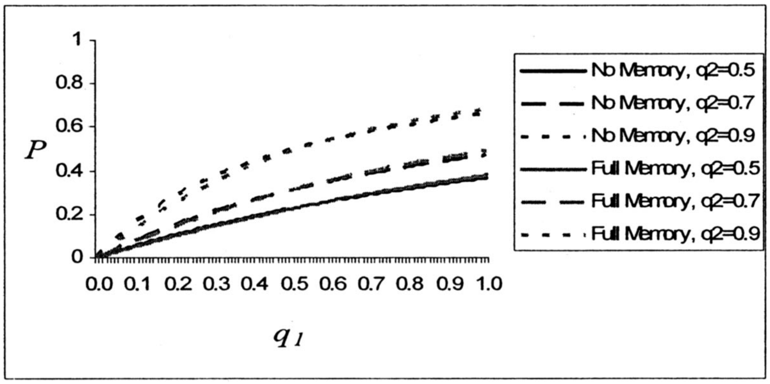


Figure 2. Probability of Acquiring a Valuable Target, $L/T = 1/4$.

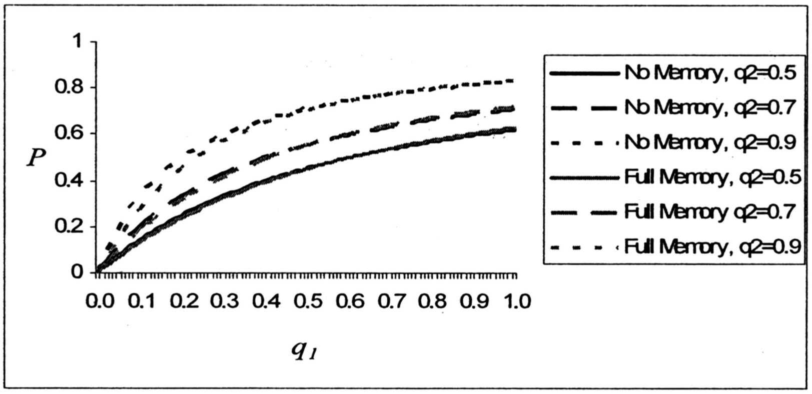


Figure 3. Probability of Acquiring a Valuable Target, $L/T = 1/2$.

example, if $q_1 = 0.8$ and $q_2 = 0.7$ then the relative differences between P_M and P_{NM}, over all twelve scenarios, range between 0% and less than 3%. As shown in Tables A1–A3 in the appendix, this conclusion remains unchanged for longer inter-detection where $\lambda = \mu$.

Based on the analysis we can conclude that, under our assumptions, memory is rather redundant design feature in UCAVs. The processing capacity on board the UCAV would be better utilized for other data processing or storing tasks. Note however that this conclusion may not be true in other tactical settings such as time-critical missions or situations where the search time is limited due to operational or logistical constraints. From now on we assume that the UCAVs have no memory.

MULTIPLE UCAVS, NO SITUATIONAL AWARENESS

In this section we explore temporal effects of the UCAVs' target engagement process. We assume no situational awareness, which means that any detected target is attacked. In other words, $q_1 = 1 - q_2 = 1$.

The probability that at time t of the engagement a certain UCAV is still searching is $e^{-(\lambda+\theta)t}$. Using conditioning, we obtain that the probability the UCAV failed by time t is:

$$Q_F(t) = \underbrace{\frac{\lambda\theta}{\mu+\theta}\int_0^t e^{-(\lambda+\theta)s}(1-e^{-(\mu+\theta)(t-s)})ds}_{\text{Probability of failure during the attack stage}}$$

$$+\underbrace{\theta\int_0^t e^{-(\lambda+\theta)s}ds}_{\substack{\text{Probability of failure}\\ \text{during the search stage}}} \qquad (4)$$

$$= \begin{cases} \dfrac{\lambda\theta}{\mu+\theta}\left[\dfrac{1-e^{-(\lambda+\theta)t}}{\lambda+\theta} - \dfrac{e^{-(\mu+\theta)t}}{\lambda-\mu}(1-e^{-(\lambda-\mu)t})\right] \\ \quad + \dfrac{\theta}{\lambda+\theta}(1-e^{-(\lambda+\theta)t}) \text{ if } \lambda \neq \mu \\ \dfrac{\lambda\theta}{\lambda+\theta}\left[\dfrac{1-e^{-(\lambda+\theta)t}}{\lambda+\theta} - te^{-(\lambda+\theta)t}\right] \\ \quad + \dfrac{\theta}{\lambda+\theta}(1-e^{-(\lambda+\theta)t}) \text{ if } \lambda = \mu \end{cases}.$$

The probability that the UCAV has completed its mission by time t without failure is

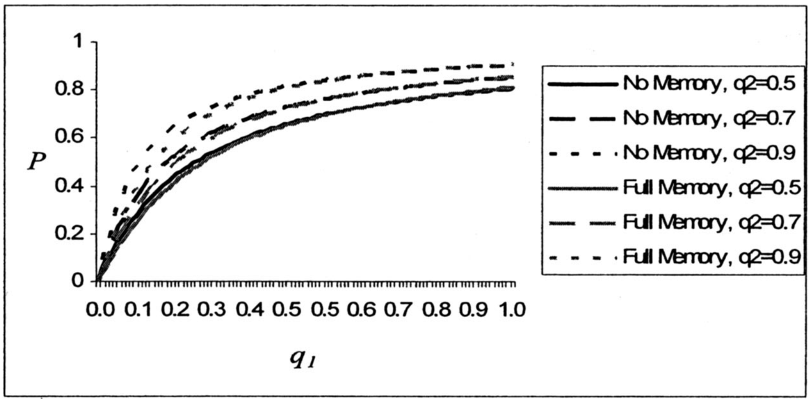


Figure 4. Probability of Acquiring a Valuable Target, $L/T = 3/4$.

$$Q_A(t) = \frac{\lambda\mu}{\mu+\theta}\int_0^t e^{-(\lambda+\theta)s}(1-e^{-(\mu+\theta)(t-s)})ds$$

$$= \begin{cases} \dfrac{\lambda\mu}{\mu+\theta} \left(\dfrac{1-e^{-(\lambda+\theta)t}}{\lambda+\theta} - \dfrac{e^{-(\mu+\theta)t}(1-e^{-(\lambda-\mu)t})}{\lambda-\mu}\right) \text{ if } \lambda \neq \mu \\ \dfrac{\lambda^2}{\lambda+\theta}\left(\dfrac{1-e^{-(\lambda+\theta)t}}{\lambda+\theta} - te^{-(\lambda+\theta)t}\right) \text{ if } \lambda = \mu, \end{cases} \tag{5}$$

and $Q_A(t) \xrightarrow[t\to\infty]{} \dfrac{\lambda\mu}{(\lambda+\theta)(\mu+\theta)}$.

Since the UCAVs are independent, the CDF of the duration of the operation is:

$$F_D(t) = [Q_F(t) + Q_A(t)]^N \tag{6}$$

and the expected number of killed targets at time t is

$$E_t = L\left(1 - \left(1 - \frac{Q_A(t)p}{T}\right)^N\right). \tag{7}$$

Consider the *base case* where the average detection time is 5 minutes, the average attack time is 30 seconds and the mean time between failures (MTBF) is 100 minutes, that is, $\lambda = 0.2$, $\mu = 2$ and $\theta = 0.01$. Figure 5 depicts the CDF of the operation completion time for various pack sizes N.

The 90th percentiles of these CDFs are 18, 23, 26, 27 and 28 minutes for packs of 4, 8, 12, 16 and 20 UCAVs, respectively. Figures 6 and 7

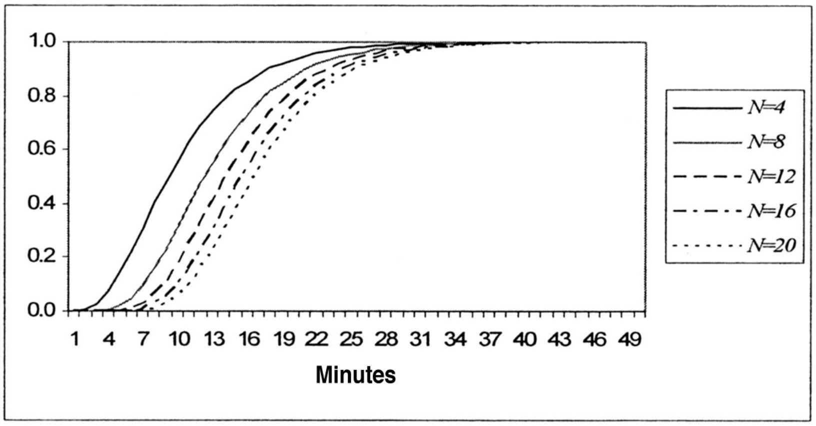


Figure 5. CDF of the Operation Completion Time for Varying N.

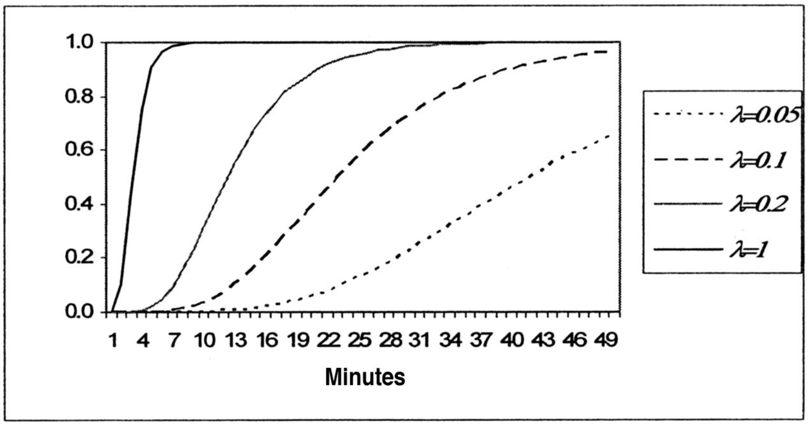


Figure 6. CDF of the Operation Completion Time for Varying λ.

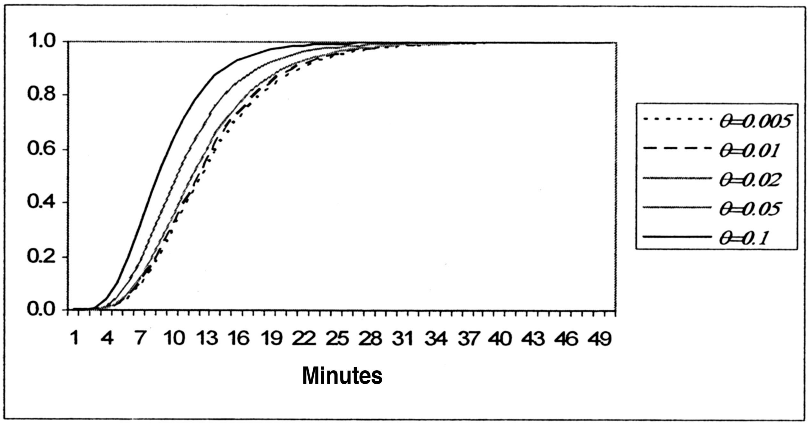


Figure 7. CDF of the Operation Completion Time for Varying θ.

present the CDF of the mission completion time for varying detection intensities (λ) and failure intensities (θ), respectively. In both cases we assume a pack of $N = 8$ UCAVs. The values of the other parameters are as in the base case.

The 90th percentiles of these distributions are 72, 40, 21, and 5 minutes for mean detection times of 20, 10, 5 and one minutes, respectively.

The 90th percentiles of the CDFs in Figure 7 are 22, 21, 20, 18 and 14 minutes for mean interception times of 200, 100, 50, 20 and 10 minutes, respectively. While the completion time of the mission is sensitive to the pack size and very sensitive to the detection intensity, it is rather insensitive to the failure rate within the relevant range. In other words, for the selected ranges of the time parameters, the most significant factor is the detection time.

Figure 8 shows the expected number of killed targets, out of an initial cluster of $L = T = 15$ targets (i.e., all targets are initially valuable), as a function of time. For λ, μ and θ we assume the base case and $N = 8$ UCAVs.

Absent situational awareness, the expected number of killed targets approaches asymptotically 3.4, 4.5 and 5.6 targets for kill probabilities .5, .7 and .9, respectively. These limit values are reached relatively fast—after about 20 minutes of operation. Figure 5.4 can help obtain some guidelines for operating the UCAVs in case they are not disposable and can be used in future operations. For example, it can identify a time t^* at which all searching UCAVs will be programmed to abandon their mission and return to the home base.

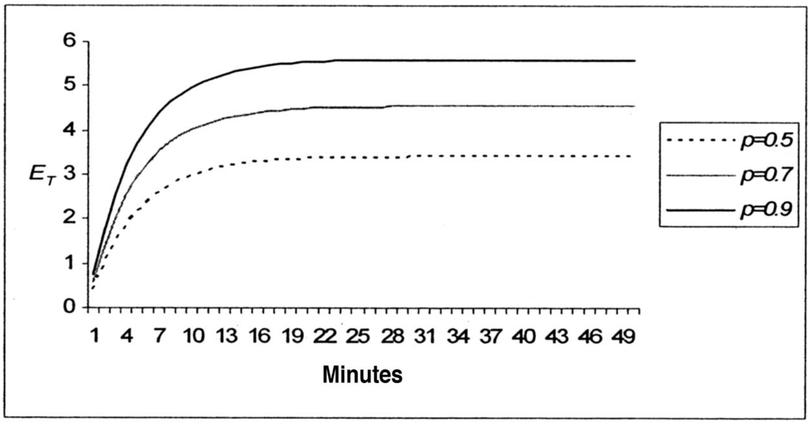


Figure 8. Expected Number of Killed Targets.

MULTIPLE UCAVS WITH IMPERFECT BDA AND LIMITED COORDINATION

Assume now that the UCAVs have limited situational awareness, that is, $0 < q_1, q_2 < 1$. Next we develop a continuous time Markov chain that represents our combat situation.

States

Let n denote the number of searching UCAVs. Initially, $n = N$. A *state* in the model is represented by $(n, m_i; i = 0, \ldots, N - n)$ where m_i indicates the number of valuable targets that are currently under attack (but have not been hit yet) by exactly i UCAVs each. An *absorbing state* in the engagement process is of the form $(0, m_0, 0, \ldots, 0)$, which means that there are no UCAVs at the search stage ($n = 0$) and no UCAVs at the attack stage. The number of valuable targets killed by the UCAVs in an absorbing state is $L - m_0$.

Example: let $L = N = 2$. There are 11 possible states: (2,2,0,0), (1,2,0,0), (1,1,1,0), (1,1,0,0), (0,2,0,0), (0,1,1,0), (0,1,0,1), (0,1,0,0), (0,0,2,0), (0,0,1,0) and (0,0,0,0). For example, the state (1,2,0,0) represents the situation where one UCAV is searching and the other UCAV is removed following a failed attack (acquired a non-valuable target or missed a valuable target or has crashed). The state (1,1,0,0) represents a similar situation, however the removed UCAV successfully acquired and killed a valuable target.

State Transitions

An event in this process is a detection, or a kill or a miss or a failure of the UCAV. A detection may lead to a change in the state if the target is identified as valuable, otherwise no change in state is recorded. A kill or a miss or a failure always results in a change of state. Figure 9 presents the possible transitions for the states in the above example ($L = N = 2$). The shaded boxes indicate absorbing (terminal) states.

In general, the following states are possible transitions from the state $(n, m_i; i = 0, \ldots, N - n)$.

(i) A searching UCAV has acquired a valuable target that is currently attacked by j other UCAVs:

$$(n - 1, m_j - 1, m_{j+1} + 1, m_i; i \neq j, j + 1) \text{ with probability } \frac{(1/T)\lambda \cdot n \cdot m_j \cdot q_1}{(\lambda + \theta) \cdot n + (\mu + \theta) \cdot \sum_{i=1}^{N-n} m_i \cdot i}. \quad (8)$$

The numerator in (8) is the rate of detection (λn) × the probability of selecting a valuable target that is currently attacked by j other UCAVs (m_j/T) × the probability of correctly identifying the valuable target (q_1).

(ii) A searching UCAV has acquired a non-valuable target or has failed (removed prematurely):

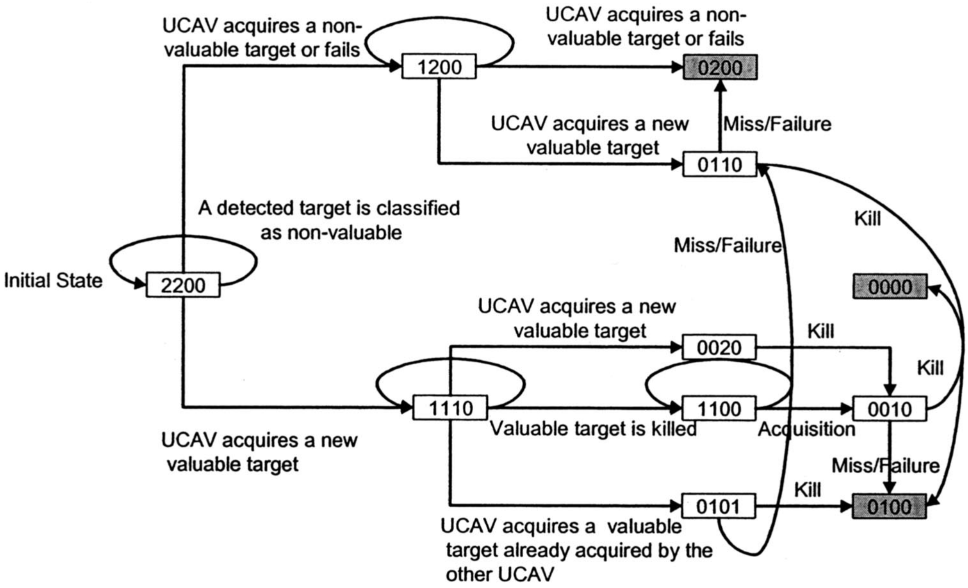


Figure 9. The State Transitions, $L = N = 2$.

$(n-1, m_i; i = 0, \ldots, N-n+1)$ with probability

$$\frac{(1/T)\cdot\lambda\cdot n\cdot\left(T-\sum_{i=0}^{N-n} m_i\right)\cdot(1-q_2)+\theta n}{(\lambda+\theta)\cdot n+(\mu+\theta)\cdot\sum_{i=1}^{N-n} m_i\cdot i}. \quad (9)$$

The numerator in (9) is the rate at which non valuable targets are acquired (= the rate of detection (λn) × the probability of selecting a non-valuable target ($(T - \Sigma_{i=0}^{N-n} m_i)/T$) × the probability of incorrectly identifying this target as valuable ($1 - q_2$)) + the failure rate of searching UCAVs (= θn).

(iii) A UCAV is the first to kill a valuable target that is currently attacked by j UCAVs:

$(n, m_j - 1; m_i; i \neq j)$ with probability

$$\frac{\mu\cdot j\cdot m_j\cdot p}{(\lambda+\theta)\cdot n+(\mu+\theta)\cdot\sum_{i=1}^{N-n} m_i\cdot i}. \quad (10)$$

The numerator in (10) is the attack rate of a single UCAV (μ) × the number of UCAVs that are attacking this type of targets ($j \cdot m_j$) × the kill probability of a single UCAV (p).

(iv) A UCAV that is attacking a valuable target, which is currently attacked by j UCAVs, is removed without completing its mission, that is, misses the target or fails during the attack:

$(n, m_{j-1}+1, m_j - 1; m_i; i \neq j-1, j)$ with probability

$$\frac{(\mu\cdot(1-p)+\theta)\cdot j\cdot m_j}{(\lambda+\theta)\cdot n+(\mu+\theta)\cdot\sum_{i=1}^{N-n} m_i\cdot i}. \quad (11)$$

The numerator in (11) is the rate of attacks that miss the target ($\mu \cdot (1-p) \cdot j \cdot m_j$, see also (10) above) + the failure rate of attacking UCAVs ($\theta \cdot j \cdot m_j$)).

(v) A detected target is classified as non-valuable and therefore passed over:

$(n, m_i; i = 0, \ldots, n)$ with probability

$$\frac{(1/T)\cdot\lambda\cdot n\cdot\left[(1-q_1)\cdot\sum_{i=0}^{N-n} m_i+q_2\cdot\left(T-\sum_{i=0}^{N-n} m_i\right)\right]}{(\lambda+\theta)\cdot n+(\mu+\theta)\cdot\sum_{i=1}^{N-n} m_i\cdot i}. \quad (12)$$

The numerator in (12) is the rate at which valuable targets are misclassified as non-valuable (= detection rate (λn) × probability of selecting a valuable target ($\sum_{i=0}^{N-n} m_i/T$) × the probability for type-1 error ($1 - q_1$)) + the rate at which non-valuable targets are classified correctly as such ($\lambda \cdot n \cdot q_2 \cdot (T - \sum_{i=0}^{N-n} m_i)/T$).

Suppose now that the UCAVs can share information and coordinate their attacks. Specifically, we assume that during the attack stage a UCAV sends out a signal that marks ("highlights") its target. The signal, which is set off when the UCAV is removed, may be received by any searching UCAV with a fixed probability r. The signals from the various UCAVs are independent. Thus, a searching UCAV that detects a target that is currently attacked by j other UCAVs avoids it without further investigation with probability $1 - (1 - r)^j$. Notice that if $r = 1$ then no incidents of multiple acquisitions (attacks) can occur. The transition rates shown above change only for cases (i) and (v):

(i) A searching UCAV has acquired a live (valuable) target that is already attacked by j other UCAVs:

$$(n - 1, m_j - 1, m_{j+1} + 1, m_i;\ i \neq j, j + 1) \text{ with probability } \frac{(1/T)\lambda \cdot n \cdot m_j \cdot q_1 \cdot (1 - r)^j}{(\lambda + \theta) \cdot n + (\mu + \theta) \cdot \sum_{i=1}^{N-n} m_i \cdot i}. \quad (13)$$

(v) A target is passed over (is valuable but recognized as being acquired by other UCAVs or is classified as non-valuable or is non-valuable):

$$(n, m_i;\ i = 0, \ldots, N - n) \text{ with probability}$$

$$\frac{(1/T) \cdot \lambda \cdot n \cdot \left[\sum_{i=0}^{N-n} m_i((1 - r)^i \cdot (1 - q_1) + 1 - (1 - r)^i) + q_2\left(T - \sum_{i=0}^{N-n} m_i\right)\right]}{(\lambda + \theta) \cdot n + (\mu + \theta) \cdot \sum_{i=1}^{N-n} m_i \cdot i}. \quad (14)$$

All other transitions ((ii)–(iv)) remain the same.

To keep the model tractable, we assume that this transfer of attack information does not apply to non-valuable targets. Otherwise we need to keep track also of the number of non-valuable targets that are being attacked by i UCAVs, which leads to a considerable expansion of the state space. If the number of non-valuable targets is relatively high compared to the numbers of valuable targets and UCAVs, and if the specificity of the sensor q_2 is reasonably high, then we can assume that instances of multiple acquisitions of non-valuable targets are highly unlikely. In particular, we assume that there are practically no incidents where a UCAV avoids acquiring a certain non-valuable target solely because it receives a signal from another UCAV that has already acquired (erroneously) that non-valuable target. Another assumption that leads to the same transition probabilities is that $r \approx 0$ for acquisitions of non-valuable targets (e.g., a UCAV realizes rather quickly that it has acquired a non-valuable target and sets the signal off immediately).

Measures of Effectiveness

To evaluate the relative effects of design and operational parameters we define four measures of effectiveness (MOE):

- *Expected relative effectiveness* (E_L) is the ratio between the expected number of killed valuable targets and their initial number. This MOE represent the effectiveness of the attack. Formally,

$$E_L = \frac{E[X]}{L} \quad (15)$$

where X is the number of killed valuable targets.

- *Expected relative efficiency* (E_N) is the ratio between the expected number of killed valuable targets and the initial number of UCAVs in the attack pack. This MOE represent how efficient is the mission. Formally,

$$E_N = \frac{E[X]}{N}. \quad (16)$$

- *Probability of attaining the mission objective* (P_α) is the probability that at least a fraction α of the L valuable targets are killed. This MOE represents tactical or operational objectives, as set by the mission commander. Clearly, this MOE is non-trivial only if $N \geq \alpha L$. Formally,

$$P_{\alpha} = \Pr(X \geq \alpha L). \tag{17}$$

In addition to the three MOEs we compute also the expected duration of a mission E_{Time}. The results are obtained by utilizing computational procedures of absorbing Markov chains (e.g., Minh (2000).

Analysis

The time parameters in our *base case* are as in Section 5: $\lambda = 0.2$, $\mu = 2$ and $\theta = 0.01$. The sensitivity, specificity and kill probabilities are $q_1 = 0.7$, $q_2 = 0.8$ and $p = 0.8$, respectively. These values represent only a reasonable reference point for the technical and operational parameters of UCAVs since most of these vehicles are still in the development phase. Even if some relevant data do exist, it would be most likely classified. Notwithstanding this limitation, the ensuing sensitivity analysis provides insights into tradeoffs among the parameters of the vehicle and the combat scenario. The base case scenario comprises a pack of $N = 8$ UCAVs that engages a total of $T = 12$ targets, out of which $L = 8$ are valuable. We first assume no coordination, that is $r = 0$. The expected number of killed valuable targets is 4.32 with engagement effectiveness and efficiency of $E_L = E_N = 0.54$. The probability of attaining the mission objective—at least 40% of the valuable targets killed—is $P_{0.4} = 0.77$. The expected duration of the operation is $E_{Time} = 30$ min. If the UCAVs are fully coordinated then $E_L = E_N = 0.55$, $P_{0.4} = 0.78$ and $E_{Time} = 30.4$ min. Clearly, in the base case, coordination has no significant effect; the changes in the MOEs values are negligible.

Next we investigate the impact of various parameters on the values of the MOEs.

(*a*) *Detection and Attack Rates*

For a fixed failure rate of $\theta = 0.01$ (base case) Figures 10–13 present the effect of the detection rate (λ) and the attack rate (μ) on the expected relative effectiveness E_N and on the probability of attaining a mission objective of 40% killed valuable targets $P_{0.4}$. Since $L = N$, the expected relative effectiveness is also the expected relative efficiency. Figures 10 and 11 apply to the case where there is no coordination among the UCAVs ($r = 0$), while Figures 12 and 13 apply to the case of full coordination ($r = 1$).

In all four charts the mean detection time of a UCAV ranges between 10 minutes ($\lambda = 0.1$) and 20 seconds ($\lambda = 3$). Both MOEs—E_N and $P_{0.4}$—are computed for four mean attack times that range from 1 minute ($\mu = 1$) to 25 seconds ($\mu = 4$). In the case of no coordination ($r = 0$), shorter attack times result in better performance of the UCAVs with respect to both MOEs. This conclusion is quite intuitive for cases of relatively high sensitivity, specificity and kill probability. Shorter attack times reduce

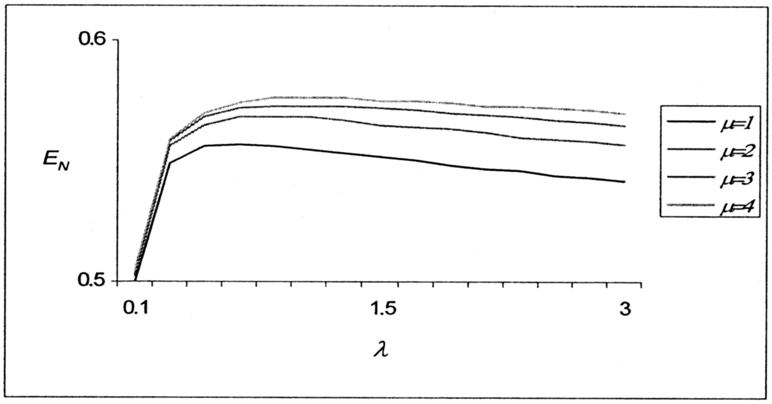


Figure 10. The Effect of Detection Rate on the Expected Relative Effectiveness (Efficiency), $r = 0$.

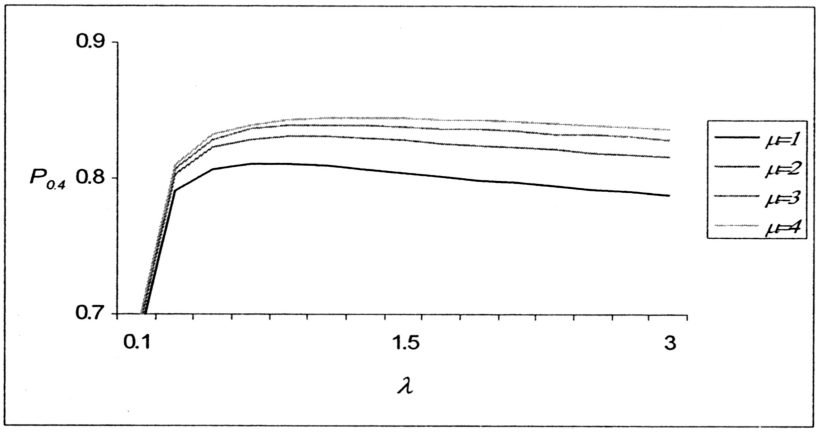


Figure 11. The Effect of Detection Rate on the Probability of Attaining 40% Killed Valuable Targets, $r = 0$.

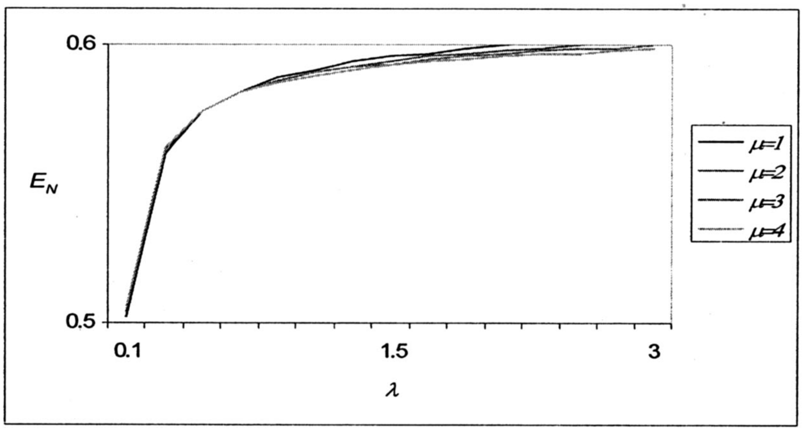


Figure 12. The Effect of Detection Rate on the Expected Relative Effectiveness (Efficiency), $r = 1$.

the possibility of redundant multiple attacks. The observation that higher detection rate may be counter-effective, as displayed by the unimodal plots in Figures 10 and 11, is less intuitive. The monotonic increasing part for small values of λ represents a race between the detection and failure processes; increasing detection rate decreases loitering time and therefore also the chances for failure. The monotonic decreasing part for larger values of λ is explained by exactly the same arguments used above for explaining the positive effect of increasing μ; shorter detection time relative to the attack time implies more opportunities for simultaneous acquisitions that lead to multiple attacks. The effect of θ is discussed later on. In the case of perfect coordination ($r = 1$) there can be no multiple acquisitions and therefore higher detection rate is always better. Since the attack time is very short compared to the MTBF (θ^{-1}) of the UCAVs, perfect coordination implies that the effect of the attack rate μ is negligible.

Note that the graphs of E_N and $P_{0.4}$ have similar shapes. For brevity we display from now on mostly results regarding E_N or E_L.

(*b*) *Sensitivity, Specificity and Kill Probability*

An interesting question regarding the UCAV's sensor capabilities is: *which property is more important, sensitivity or specificity?* Recall that higher sensitivity means lower probability for type I error (misclassifying a valuable target), while higher specificity implies lower probability for type II error (misclassifying a non valuable target). Figures 14 and 15 show the effect of changing the sensitivity and specificity of the sensor, respectively. The results

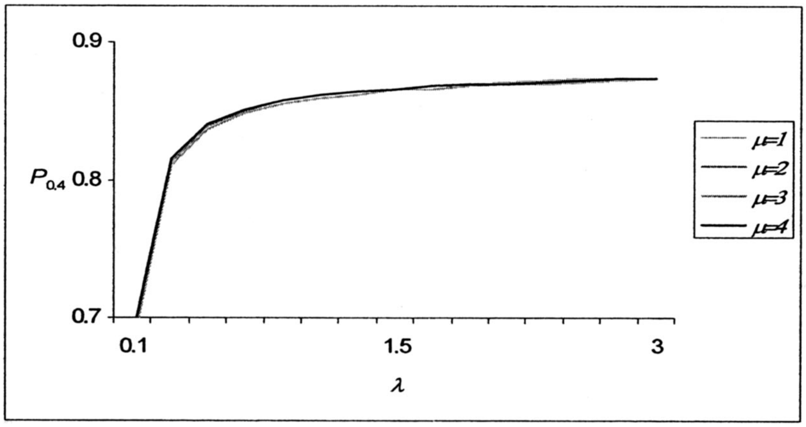


Figure 13. The Effect of Detection Rate on the Probability of Attaining 40% Killed Valuable Targets, $r = 1$.

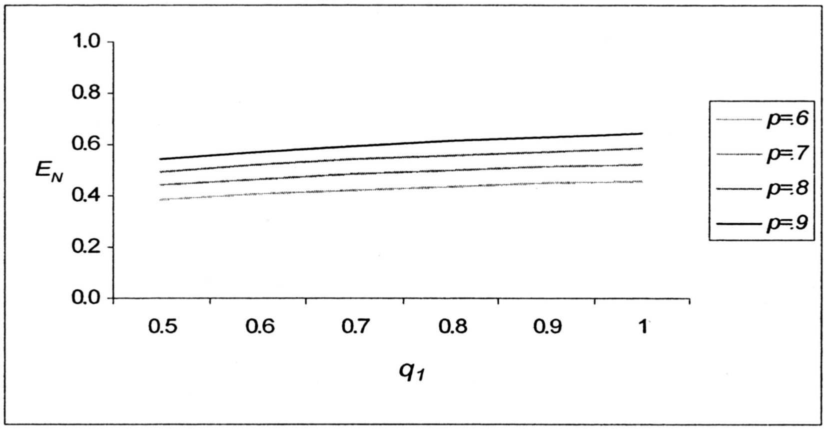


Figure 14. The Effect of Sensor Sensitivity on the Expected Relative Effectiveness.

are displayed for 4 values of kill probability: 0.6, 0.7, 0.8 and 0.9. All other parameters are set at their base case values.

Note that while E_N is a concave function of the sensor's sensitivity, it is a convex function of its specificity. When we move q_1 from 0.5 to 1, E_N increases by 18% for all values of p. The corresponding increase in E_N when q_2 varies is 47% for $p = 0.6$ and 53% for $p = 0.9$. We conclude that the effect of specificity on the outcome of the attack is stronger than sensitivity, and this effect becomes more significant for higher values of kill probability. Specifically, suppose that the decision is either to increase the sensitivity of the sensor by 20% from its current base case value, or to increase by a similar rate its specificity. Recall that for the base case $E_N = 0.54$. If q_1 is increased by 20% then $E_N = 0.56$, while if q_2 is increased by 20% then $E_N = 0.64$. The choice is clear; in order to increase the effectiveness of the attack one should invest in improving the specificity of the UCAV's sensor, rather than its sensitivity. This conclusion applies to our case of disposable (or one-weapon) UCAVs where a false positive error is irreversible. This may not be the case if the UCAV has multiple weapons and the mission is not time-critical. The recommendation to invest in better specificity is enhanced by other measures of merit such as the human and political cost of attacking a wrong target (e.g., the bombing of the Chinese embassy in Belgrade by NATO forces in 1999).

(*c*) *Failure Rate and Coordination*

Arguably, UCAVs' coordination can be effective only if the attack stage is long compared

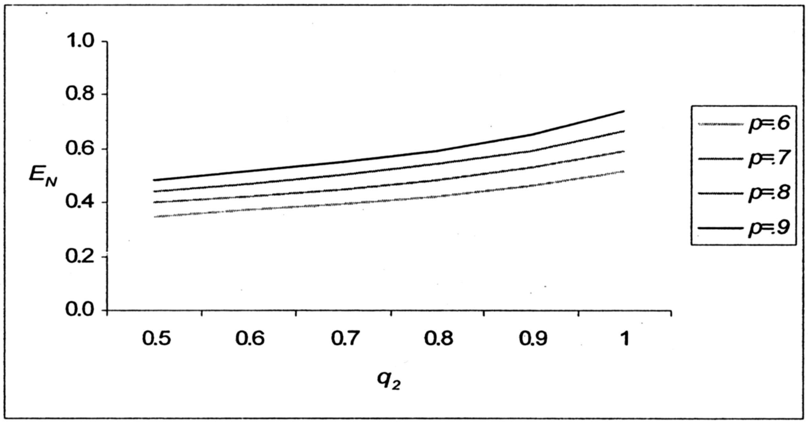


Figure 15. The Effect of Sensor Specificity on the Expected Relative Effectiveness.

to the search stage. If it is short, then multiple acquisitions are very unlikely and therefore there is no practical need for coordination. It is shown next that the failure rate may affect the benefit the pack gains from coordination. Let $L = 6$, and suppose that the attack time is four time shorter than the detection time, which is set at its base case value. This situation may represent a standoff attack. The failure rate ranges between 0 (no failure during the mission) to 0.1 (MTBF = 10 min). Figure 16 presents the expected relative effectiveness for $r = 0$ and $r = 1$. All other parameters are set at their base case values.

As one would expect, the effectiveness of the UCAVs decreases as the failure rate increases. Note that even in this extreme scenario, where conditions are relatively favorable for effective coordination, the effect is minute. Moreover, while for smaller failure rates full coordination is somewhat more effective than no coordination, the opposite is true for larger failure rates for which coordination actually reduces the mission effectiveness. The latter counter-intuitive observation is due to the fact that if UCAVs pass over targets, they prolong their stay in the target area and therefore increase their chances to be intercepted before staging their attack.

Another way to avoid multiple acquisitions is to employ the UCAVs sequentially rather than simultaneously as a pack. This tactical solution to multiple acquisition problem leads to a different Markov model that is based on the probabilities given in (2) above. Taking once again $L = 6$, $\mu = \lambda/4$ and the rest of the pa-

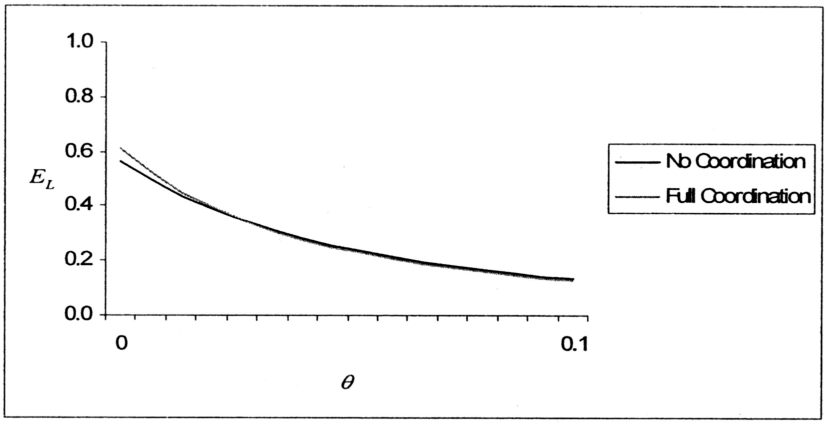


Figure 16. The Effect of Failure Rate and Coordination on the Expected Relative Effectiveness.

rameters at their base case values, Figure 17 presents the value of the expected relative effectiveness E_L for three cases: No coordination, perfect coordination and sequential engagement. These values are computed as functions of the specificity probability q_2.

For poor to moderate specificity the three graphs coincide. For high specificity, eliminating multiple acquisitions, either by a design features (coordination) or tactics (sequential engagement) has some effect. The effect is similar in both cases, with a slight advantage to the tactical solution, which is applicable only for non time-critical targets.

(*d*) *Scenario Parameters*

So far we have analyzed the effect of parameters that are associated with the design of the UCAVs. Figures 18 and 19 display the effect of the scenario. Figure 18 presents the value of E_L when the number of valuable targets L and the specificity probability q_2 vary in the target area. Note that besides being a design parameter, specificity is also a scenario parameter that may depend on the clutter in the target area. Figure 19 displays the combined effect of L and the number of UCAVs N. The MOE here is $P_{0.4}$, which represents a specific tactical objective.

From Figure 18 we see once again the effect of specificity. At low specificity, the rate of killed valuable targets is relatively insensitive to their number. At high specificity this rate decreases with the number of targets, as one would expect.

Figure 19 examines the impact of the number of valuable targets on the engagement performance from another angle. For small number of UCAVs the 40% attrition probability is very sensitive to the number of targets. This sensitivity diminishes as N gets larger. Note that Figure 19 may be used also as a decision support tool for mission planning. For example, if there are four valuable targets in the target area, then in order to attain the mission objective—two killed targets—with probability of at least 0.8, then the pack must contain at least seven UCAVs. This can be seen by observing the point at which the graph corresponding to $L = 4$ crosses the 0.8 threshold.

SUMMARY, CONCLUSIONS AND FUTURE RESEARCH

In this paper we explore several design and operational aspects of employing a pack of autonomous UCAVs against valuable targets that are imbedded among other, non-valuable targets. Utilizing newly developed analytic probability models, we evaluate the effect of key design and operational parameters on the performance of the pack. First, it is shown that under reasonable assumptions *memory* is a redundant property. The processing capacity in the UCAV *brain* should be utilized to other tasks such as enhanced recognition capability. Second, based on a transient model, inter-temporal behavior of the system is explored and some insights regarding mission duration and maximum allowable loitering time are obtained. It is shown that detection rate is a major

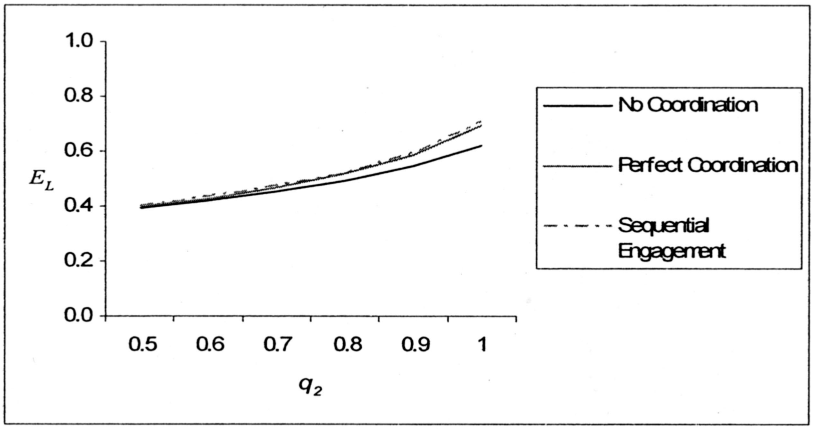


Figure 17. The Effect of Eliminating Multiple Acquisitions.

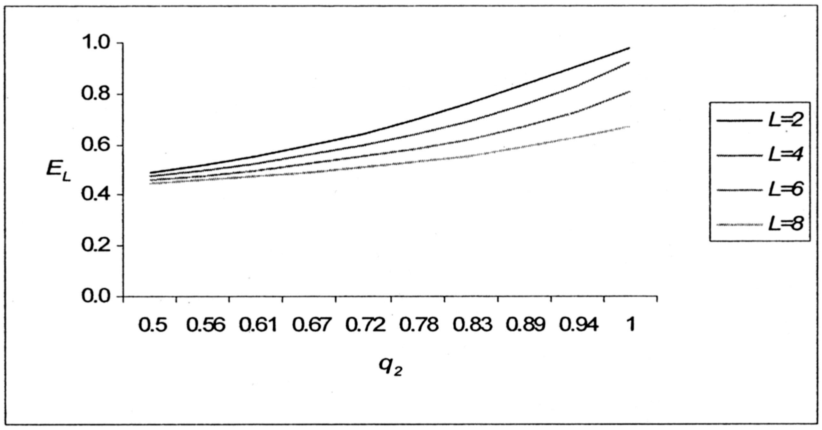


Figure 18. The Effect of the Number of Valuable Targets.

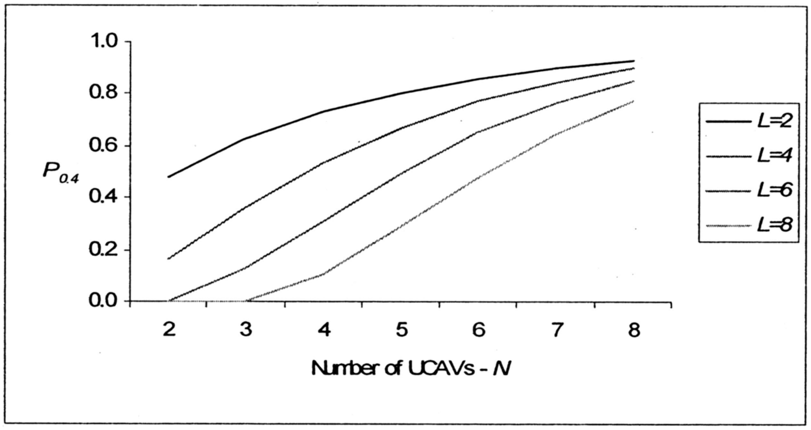


Figure 19. Probability of Attaining 40% Killed Valuable Targets as a Function of *M* and *N*.

factor in determining the duration of the operation. Finally, in Section 6 we implement a large-scale continuous-time Markov model to analyze the effect of weapon coordination on multiple acquisitions, and the effect of BDA on multiple kills. The two main conclusions from the analysis are: (1) attack coordination among UCAVs is largely an insignificant feature for the scenarios analyzed, and (2) specificity of the UCAV's sensor is more important than its sensitivity. The first conclusion is true as long as the valuable targets are homogeneous. It essentially says that the random uniform and independent selection is the right thing to do when engaging uniform targets. If among the valuable targets there are some that are more noticeable or attractive then targeting coordination may improve the engagement performance. The case of non-homogeneous targets is left for future research. The second conclusion tells us that avoiding non-valuable targets is more beneficial than picking correctly valuable ones. This observation, which at first glance may look a little odd, is quite logical. Type I error by a UCAV (passing over a valuable target) can be rectified later on. Type II error (acquiring and attacking non-valuable target) cannot.

The models described in this paper are limited to homogeneous targets, homogeneous UCAVs and to the engagement rules specified. Another limitation is the assumption that all the temporal random variables are exponential. While this assumption is reasonable for the failure and detection processes, the attack time is probably not well represented by a constant failure-rate (CFR) distribution. Accordingly, the models presented in this paper may be extended to account for non-homogeneous targets, multiple types of UCAVs and more general time CDFs (e.g., non-exponential attack times). Another interesting and potentially important extension is to incorporate in the models decision rules where the UCAVs manifest some level of cognitive capability. Specifically, in reality both sensitivity and specificity probabilities may depend on the time a UCAV spends investigating a target. This aspect is not captured in our models and may lead to interesting optimization models.

Acknowledgement

This research was partially supported by the Navy Modeling and Simulation Office, Assistant Secretary of the Navy for Research, Development and Acquisition.

REFERENCES

Airforce Technology, 2005, "Predator RQ-1/MQ-1/MQ-9 Unmanned Aerial Vehicle (UAV), USA", *http://www.airforce-technology.com/projects/predator/.*

Aviv, Y., and Kress, M., 1997, "Evaluating the Effectiveness of Shoot-Look-Shoot Tactics in the Presence of Incomplete Damage Information", *Military Operations Research*, V. 3, No. 1, pp 79−89.

Chandler, P. R., Pachter, M., Nygard, K. E., and Swaroop, D., 2002, "Cooperative Control for Target Classification", in R. Murphy and P. Pardalos, Ed. *Cooperative Control and Optimization*, Kluwer Academic Publishers, 2002, pp 1–19.

Gillen, D., and Jacques, D. R., 2002, "Cooperative Behavior Schemes for Improving the Effectiveness of Autonomous Wide Area Search Munitions", Kluwer Academic Publishers, 2002, pp 95–120.

Glazebrook, K., and Washburn, A. R., 2004, "Shoot-Look-Shoot: a Review and Extension", *Operations Research*, V. 52, pp 454−463.

Jacques, D. R., 2002, "Modeling Considerations for Wide Area Search Munition Effectiveness Analysis", *Proceedings of the 2002 Winter Simulation Conference*, pp 878−886.

Jane's, 2000a, "Briefing—Autonomous Weapons", *Jane's Defense Weekly*, V. 33/6, February 9, 2000.

Jane's, 2000b, "Germany Funds Final Phase of TAIFUN UAV", *Jane's Defense Weekly*, V. 33/17, April 26, 2000.

Jane's, 2002, "Air-to-Surface Missiles", *Jane's Air-Launched Weapons*, V. 40, 23 April, 2002.

Jeffcoat, D., "Coupled Detection Rates: An Introduction," in *Theory and Algorithms for Cooperative Systems*, Grundel, D., Murphey, R., and Pardalis, P., editors, New Jersey, World Scientific Publishing, 2004, pp 157–167.

Kress, M., 1991, "A Two-on-One Stochastic Duel with Maneuvering and Fire Allocation Tactics", *Naval Research Logistics*, V. 38, No. 3, pp 303–313.

Kress, M., and Talmor, I., 1999, "A New Look at the 3:1 Rule of Combat Through Markov Stochastic Lanchester Models", *Journal of the Operational Research Society*, V. 50, No. 7, pp 733–744.

Lockheed Martin, 2004, "Autonomous Wide Area Search Munition", *Defense Update—International Online Defense Magazine*, Issue 5, 2004, *http://www.defense-update.com/products/a/awasm.htm.*

Manor, G., and Kress, M., 1997, "Optimality of the Greedy Shooting Strategy in the Presence of Incomplete Damage Information", *Naval Research Logistics*, V. 44, No. 7, pp 613−622.

Minh, D. L., 2000, *Applied Probability Models*, Duxbury Press, pp 98−107.

Richards, A., Bellingham, J., Tillerson, M., and How, J., 2002, "Co-ordination and Control of Multiple UAVs", Proceedings of the AIAA *Guidance, Navigation and Control Conference*, Monterey CA.

Yost, K., and Washburn, A. R., 2000, "Optimizing assignment of Air-to-Ground Assets and BDA Sensors", *Military Operations Research*, V. 5, No. 2, pp 77–91.

APPENDIX

Acquisition probabilities in the full-memory (M) and no-memory (NM) cases.

Table A1. Acquisition Probability—Proportion of Valuable Targets = 1/4

q_1	q_2	$\frac{\lambda}{\theta} = 20$				$\frac{\lambda}{\theta} = 3$			
		$\mu = \lambda$		$\mu = 10\lambda$		$\mu = \lambda$		$\mu = 10\lambda$	
		NM	*M*	*NM*	*M*	*NM*	*M*	*NM*	*M*
0.5	0.5	.22	.22	.23	.23	.11	.11	.15	.14
	0.7	.30	.30	.31	.31	.14	.14	.18	.17
	0.9	.48	.47	.50	.49	.18	.17	.23	.22
0.7	0.5	.28	.28	.29	.29	.15	.15	.19	.19
	0.7	.37	.38	.39	.40	.18	.18	.23	.23
	0.9	.56	.56	.58	.59	.23	.22	.29	.29
0.9	0.5	.33	.34	.34	.35	.18	.18	.23	.24
	0.7	.43	.44	.45	.46	.22	.22	.28	.28
	0.9	.61	.63	.64	.66	.27	.27	.34	.35

Table A2. Acquisition Probability—Proportion of Valuable Targets = 1/2

q_1	q_2	$\frac{\lambda}{\theta} = 20$				$\frac{\lambda}{\theta} = 3$			
		$\mu = \lambda$		$\mu = 10\lambda$		$\mu = \lambda$		$\mu = 10\lambda$	
		NM	*M*	*NM*	*M*	*NM*	*M*	*NM*	*M*
0.5	0.5	.43	.43	.45	.45	.23	.22	.29	.29
	0.7	.53	.53	.55	.55	.26	.25	.33	.32
	0.9	.68	.67	.71	.71	.30	.29	.38	.37
0.7	0.5	.51	.52	.54	.54	.28	.28	.36	.36
	0.7	.61	.61	.63	.64	.31	.31	.46	.46
	0.9	.74	.74	.77	.78	.36	.36	.49	.49
0.9	0.5	.57	.58	.60	.60	.33	.33	.45	.45
	0.7	.68	.69	.69	.70	.36	.36	.49	.50
	0.9	.79	.80	.81	.82	.41	.41	.55	.55

Table A3. Acquisition Probability—Proportion of Valuable Targets = 3/4

q_1	q_2	$\frac{\lambda}{\theta} = 20$				$\frac{\lambda}{\theta} = 3$			
		$\mu = \lambda$		$\mu = 10\lambda$		$\mu = \lambda$		$\mu = 10\lambda$	
		NM	*M*	*NM*	*M*	*NM*	*M*	*NM*	*M*
0.5	0.5	.65	.65	.68	.67	.34	.33	.44	.43
	0.7	.71	.71	.75	.74	.36	.35	.46	.46
	0.9	.79	.79	.83	.82	.38	.38	.50	.49
0.7	0.5	.71	.72	.75	.75	.40	.40	.52	.51
	0.7	.77	.77	.80	.81	.42	.42	.54	.54
	0.9	.83	.83	.87	.87	.45	.44	.58	.57
0.9	0.5	.76	.76	.79	.80	.45	.45	.58	.58
	0.7	.80	.81	.84	.84	.47	.47	.60	.60
	0.9	.86	.86	.90	.90	.49	.49	.63	.63

Distorted Risk Measures with Application to Military Capability Shortfalls

Maj Edwin J. Offut

Air Force Institute of Technology
edwin.offut@afit.edu

Dr. Jeffrey P. Kharoufeh

Air Force Institute of Technology
Jeffrey.kharoufeh@afit.edu

Dr. Richard F. Deckro

Air Force Institute of Technology
richard.deckro@afit.edu

APPLICATION AREA(S):
Readiness
OR METHODOLOGIES:
Applied Statistics and Linear Programming

ABSTRACT

We consider the problem of selecting an appropriate distortion function and associated parameters to account for rare but catastrophic events that may result from a shortfall of military or security capabilities. Additionally, we describe the means by which a decision maker may allocate resources among various risk-mitigating systems, subject to a finite budget constraint, while considering the risk of such shortfalls. Through a numerical illustration, we show that the optimal allocation of resources is sensitive to the decision maker's level of risk aversion.

Keywords: Distortion function, risk measures, capability.

INTRODUCTION

In this paper, we review and illustrate the concept of *distorted* risk measures in order to analyze the risk of shortfalls in military capabilities. The application of distorted risk measures facilitates a method for optimally allocating resources among various military capabilities while taking into account the risk of capability shortfalls. However, an open and difficult question is the selection of appropriate distortion functions (and parameters), and their interpretations, for a given risk scenario. In this study, we propose numerical measures that can be used to assist in distortion function selection. Moreover, it is our aim to elucidate the usefulness of distorted risk measures, especially for scenarios involving low-likelihood, catastrophic events.

Consider a military or national-level decision maker who is faced with addressing shortfalls in military or homeland security capability. However, due to budgetary (or possibly other) constraints, only a subset of shortfalls can be addressed. We assume that input data from subject matter experts, in the form of risks of capability shortfalls, can be converted into appropriate risk distributions using, for example, techniques such as those outlined in Clemen and Reilly (2001) for assessing continuous or discrete probabilities. While the decision maker may trust and value the opinions of subject matter experts, he or she may desire to assign their own risk priorities in the resource allocation process to reflect additional information and/or considerations not necessarily available to the subject matter experts.

Distortion functions can be used to alter standard risk measures for scenarios in which low-likelihood, yet potentially catastrophic, occurrences in the tail of the risk distribution are of interest but are often suppressed by standard risk measures (e.g., expectation and conditional expectation). In such cases, distortion functions serve the purpose of shifting probability density toward the region of the distribution that corresponds to highly adverse outcomes, thereby inflating the expectation risk measure. A wide variety of shaping effects and degrees of effect are possible depending on the distortion function selected and its parameters. The challenge for the decision maker is to select appropriate distortion functions to apply to risk distributions suggested by his or her subordinates. Similarly, because the degree of distortion applied via the parameter selection can be (directly or indirectly) linked to the decision maker's degree of risk aversion, the selection of appropriate distortion function parameters must also be considered. While numerous distortion functions have been introduced in the mathematical finance and insurance literature (see McLeish and Reesor 2003 and Wang 1996a), there is not a universally accepted, formal methodology for the selection of a distortion function or its associated parameters.

As a relatively new and competing theory for the pricing of risk (prospect theory is the other), the properties of parametric distortion functions have been examined in the finance and insurance literature for the past ten years. The seminal work on distortion functions is due to Wang (1995), who first proposed transforming the survivor function of the risk using the proportional hazard transform. Subsequently, Wang (1996a) generalized the theory of distortion to an entire class of functions used to calculate insurance premiums. Wang *et al.* (1997) provided an axiomatic theory of insurance premiums pricing. Distortion and the axiomatic theory are closely tied to the concept of risk measure *coherency*, outlined by Artzner *et al.* (1997) and further developed by the same authors in Artzner *et al.* (1999). Simply stated, a coherent risk measure is one that accurately portrays the way financial markets operate. Artzner *et al.* (1997) establish four attributes that a coherent risk measure must possess, the most important being that of subadditivity which means that the aggregation of several risks should not increase the overall risk. McLeish and Reesor (2003) proved that a concave distortion function produces a coherent risk measure. Wirch and Hardy (1999) made two general observations regarding distortion parameters. First, they associate the parameters with a decision maker's risk aversion level toward risk in the far right tail of the distribution. Second, they state that the selection of distortion parameters is mostly a "political" decision. To our knowledge, the problem of selecting an appropriate distortion function and its parameter(s) has not been formally addressed in the risk analysis literature.

In this paper we seek to provide some guidance for the selection of distortion functions and their parameters when concerned with inflating the right tail of a risk distribution. We are motivated by the potentially catastrophic losses that may result from military or homeland security capability shortfalls. Specifically, this concerns the representation of catastrophic risks that cannot, for operational or political reasons, be disregarded despite their low likelihood of occurrence. Throughout this study, the focus of our attention is on the expected value of the risk; however, it is worth noting that we may choose any other coherent risk measure. Though it is admittedly difficult to generalize the guidelines to an arbitrary scenario, we attempt to provide a framework within which the risk of military capability shortfalls may be considered. For other contexts (e.g., insurance or financial risk), it may be necessary to vary the framework or even develop a separate analysis.

We summarize and study the impact of three of the most widely referenced distortion functions on four parametric probability distributions, specifically the exponential, Weibull, triangular, and uniform distributions. Whenever possible, we provide closed-form expressions for the corresponding distorted risk measures. However, when such expressions are not available, it is still possible to compute the measures by numerical methods. We propose two simple measures of distortion effects, *effectiveness* and *efficiency*, and by means of a simple designed experiment, we argue that some distortions may be preferable to others, depending on the risk distribution and the extent to which distortion is desired. Finally, we illustrate the means by which a decision maker's risk aversion levels may be incorporated into a resource allocation problem using appropriate distortion selection. The results of the study offer some practical guidance for the application of distortion functions to some specific risk scenarios.

The remainder of this paper is organized as follows. In the next section, we define and review the concept of distorted risk measures and provide analytical results for the distorted expectation risk measure using a few parametric probability distributions. In the Measuring Distortion Effects section we propose two distortion performance measures and use a simple designed experiment to help establish some guidelines for distortion function selection. Resource Allocation and Distortion: An Illustration presents an illustrative example of a resource allocation problem which considers the decision maker's risk aversion levels while the Conclusion provides a few closing remarks and future research directions.

A REVIEW OF DISTORTION FUNCTIONS

This section provides a brief overview of distortion functions and coherent risk measures. Before presenting mathematical descriptions, we first provide an intuitive motivation for the use of distortions in a given risk scenario.

Concept of Distortion

Assume that risk is a nonnegative random variable. If one is concerned only about the probability that the random variable is above (or below) some critical value, and not about what happens above that value, then it is instructive to use a simple quantile risk measure (i.e., the Value-at-Risk (VaR) measure in finance). In such a case, the distortion function is simply a step function and the resulting distorted risk measure has no probability in the tail of the original risk distribution. In many cases, however, the tail of the distribution is of interest because unacceptable, highly catastrophic losses can occur with low probability. In these scenarios, it makes sense to amplify the probability in the region of the original risk distribution that corresponds to highly adverse and unacceptable outcomes. In our context, a military or national-level decision maker may forego the opportunity to acquire certain military capabilities. The risks involved in a shortfall of military capability can be viewed as the potential for loss of human life, loss of assets, or other significant losses.

We assume that a nonnegative risk X is defined on an appropriate probability space (Ω, $\mathcal{F}$, P) with cumulative distribution function (c.d.f.) given by $F(x) = P(X \leq x)$, $x \geq 0$ and survivor function $S(x) \equiv P(X > x)$, $x \geq 0$. The expectation risk measure is given by

$$E(X) = \int_0^\infty S(x)dx. \tag{1}$$

The problem of selecting the risk distribution for extreme events is a difficult question in its own right. We do not specifically address distribution selection here; however, some guidance is given by Lambert *et al.* (1994). The objective of a distortion function is to transform the survivor function $S(x)$ so that when a risk measure is computed, the resulting distorted measure more adequately reflects the possibility and impact of extreme events. More formally, the distortion of S is given by the composition function

$$g(S(x)) \equiv (g \circ S)(x), \tag{2}$$

where g is a function satisfying (see Wirch and Hardy 1999):

1. $g : [0, 1] \to [0, 1]$ is monotonically increasing;
2. $\lim_{u \downarrow 0} g(u) = 0$; and
3. $\lim_{u \uparrow 1} g(u) = 1$.

The function $\hat{S}(x) \equiv (g \circ S)(x)$ is again a survivor function with the usual properties: its range is $[0, 1]$, it is non-increasing in x, and integrating over the range of X gives the (distorted) expectation. That is, under the distortion g, the expectation risk measure is

$$\hat{E}(X) = \int_0^\infty \hat{S}(x)dx = \int_0^\infty (g \circ S)(x)dx. \tag{3}$$

The concept of distortion is closely tied to the concept of risk measure coherency which was formalized by Artzner *et al.* (1999). Suppose X and Y are two nonnegative random variables representing two risks and let ρ denote a risk measure. Then ρ is said to be a coherent risk measure if it satisfies the following four axioms:

1. Translation invariance: For all real α and r, $\rho(X + \alpha r) = \rho(X) - \alpha$
2. Subadditivity: $\rho(X + Y) \leq \rho(X) + \rho(Y)$
3. Positive homogeneity: For all $\alpha \geq 0$, $\rho(\alpha X) = \alpha\rho(X)$
4. Monotonicity: If $X \leq Y$, then $\rho(X) \geq \rho(Y)$

McLeish and Reesor (2003) have shown that, if g is a concave function, then the resulting distorted risk measure will satisfy the four axioms of coherency. This fact will be useful in determining appropriate distortion parameter values since we seek a resulting risk measure that is coherent. Next, we review some of the most commonly applied distortions.

Common Distortion Functions

The distortion functions most frequently encountered in the literature are the gamma-beta distortion and its variants which are discussed extensively in McLeish and Reesor (2003). This family of distortion functions consists of the gamma-beta, beta, proportional hazard (PH), dual power (DP), gamma, and exponential (EX) distortions. Among these six, the single-parameter distortions (PH, DP, and EX) will be considered for three primary reasons: i) the effects of an individual parameter may be observed more easily; ii) the distorted expectation risk measure can be computed analytically in many cases, and numerically in others; and iii) it is desirable to minimize the number of parameters that need to be estimated.

The gamma-beta distortion is defined as

$$g_{GB}(S(x)) = \int_0^{S(x)} Kt^{a-1}(1-t)^{b-1}\exp(-t/c)dt, \tag{4}$$

where

$$K^{-1} = \int_0^1 t^{a-1}(1-t)^{b-1}\exp(-t/c)dt.$$

This distortion serves as the basis for other distortions when we assume certain values for the parameters a, b, and c. It is worth mentioning here that a, b, and c may assume any nonnegative values; however, McLeish and Reesor (2003) have shown that $0 \le a \le 1$, $b \ge 1$, and $c \ge 0$ are sufficient to ensure concavity of the distortion function, and thus coherency of the associated risk measure.

By setting $b = 1$ and allowing $c \to \infty$ in (4), we obtain the proportional hazard (PH) distortion given by

$$(g_{PH} \circ S)(x) = S^a(x),\ 0 \le a \le 1. \tag{5}$$

The attractive feature of (5) is its ease of computation. By setting $a = 1$ and allowing $c \to \infty$ in (4), we arrive at the dual power (DP) distortion given by

$$(g_{DP} \circ S)(x) = 1 - (1 - S(x))^b,\ b \ge 1. \tag{6}$$

As noted by Wirch and Hardy (1999), this distortion has perhaps the most lucid interpretation. For an integer value of b, the expectation risk measure corresponds to the expected value of the maximum of a sample of b observations of X. Finally, the exponential (EX) distortion depends only on the single parameter c and is given by

$$(g_{EX} \circ S)(x) = \frac{1 - e^{-S(x)/c}}{1 - e^{-1/c}},\ c \ge 0. \tag{7}$$

This distortion corresponds to an exponential random variable restricted to the interval [0, 1].

The question with which we concern ourselves in this study is, "Why may one of the distortion functions be preferable to the others in a given context?" The answer to this question is that it depends upon the risk scenario under consideration. In Figure 1, each of the single-parameter distortions is applied to an exponentially distributed risk X with rate parameter $\lambda = 3.5$. The gamma-beta distortion is also included since it uses all three parameters a, b, and c. The undistorted exponential density is depicted by the solid line. Among the single-parameter distortions, the proportional hazard (PH) distortion has the greatest effect on the right tail of the distribution, thickening it considerably. The dual power (DP) distortion, while inflating the right tail slightly, has a much more noticeable effect on the left side of the distribution, shifting the mode away from zero. The exponential (EX) distortion can best be described as a combination of the effects of the PH and DP. In general, these effects are consistent for the other distributions considered in this paper.

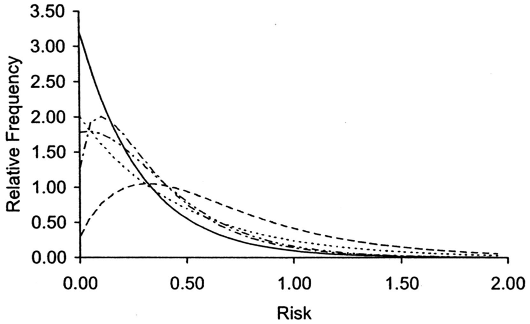


Figure 1. Distorted density when $X \sim$ Exp(3.5) with distortion parameters $a = 0.6$, $b = 1.5$, and $c = 0.8$ for GB (solid is no distortion, – – – GB, · · · PH, – · – · DP, – · · · – EX).

From this plot, we can clearly observe why a particular distortion function might be preferred over another. One may be more concerned with inflating the right tail rather than altering the left-hand side of the distribution. In what follows, we compute the distorted expectation risk measure using the three single-parameter distortion functions and four parametric probability distributions. These will be used to study the effects of distortions and their parameters in the Measuring Distortion Effects section.

Computing Distorted Measures

In order to elucidate the effects of distortion, we now apply the single-parameter distortions to a set of parametric probability distributions. The four distributions we consider are: exponential with rate parameter λ (denoted Exp(λ)); Weibull with shape parameter β and scale parameter θ (denoted Weib(β, θ)); triangular on the closed interval $[\theta_1, \theta_2]$ with mode m (denoted Tria(θ_1, θ_2, m)); and continuous uniform on the closed interval $[\theta_1, \theta_2]$ (denoted $U(\theta_1, \theta_2)$). We select these four distributions because they are representative of risk distributions from a variety of disciplines including actuarial science, financial and insurance risk, as well as reliability. Moreover, they span a range of distribution shapes on both bounded and unbounded intervals. Finally, they have a relatively small number of parameters that may be easily estimated using information that is likely to be available from subject matter experts.

For each combination of distribution and distortion, we have attempted to summarize closed-form expressions for the distorted expectation risk measure given by equation (3). In some cases, explicit expressions are attainable, while others remain as definite or indefinite integrals that can be evaluated numerically using standard methods. These results are recorded in Tables 1 through 4.

First suppose the risk X is exponentially distributed with rate parameter $\lambda > 0$. In such case, the survivor function is given by

$$S(x) = \begin{cases} e^{-\lambda x} & \text{if } x \geq 0, \lambda > 0 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

The undistorted risk measure is $\mu_0 \equiv E(X) = \lambda^{-1}$. Table 1 provides a summary of the distorted survivor function and the distorted risk measure computed by equation (3).

Table 1. Distorted risk measures when $X \sim$ Exp(λ)

Distortion function	$\hat{S}(x)$	$\hat{E}[X]$
g_{PH}	$e^{-\lambda a x}$	$(\lambda a)^{-1}$
g_{DP}	$1 - (1 - e^{-\lambda x})^b$	$\int_0^\infty [1 - (1 - e^{-\lambda x})^b]\, dx$
g_{EX}	$\dfrac{1 - \exp(-e^{-\lambda x}/c)}{1 - \exp(-1/c)}$	$\int_0^\infty \dfrac{1 - \exp(-e^{-\lambda x}/c)}{1 - \exp(-1/c)}\, dx$

Next, suppose the risk follows a Weibull distribution with parameters β and θ. In such case, the survivor function is

$$S(x) = \begin{cases} \exp((-x/\theta)^{\beta}) & \text{if } x \geq 0, \beta > 0, \theta > 0 \\ 0 & \text{otherwise} \end{cases}, \tag{9}$$

and the undistorted expectation is $\mu_0 = (\theta/\beta)\Gamma(\beta^{-1})$, where $\Gamma(\cdot)$ is the gamma function. Similarly, the distorted risk measures are summarized in Table 2.

The third distribution we considered was the triangular distribution on $[\theta_1, \theta_2]$ with mode value m. The survivor function is given by

$$S(x) = \begin{cases} 1 & \text{if } x < \theta_1 \\ 1 - \dfrac{(x - \theta_1)^2}{(\theta_2 - \theta_1)(m - \theta_1)} & \text{if } \theta_1 \leq x \leq m \\ \dfrac{(\theta_2 - x)^2}{(\theta_2 - \theta_1)(\theta_2 - m)} & \text{if } m < x \leq \theta_2 \\ 0 & \text{if } x > \theta_2 \end{cases}, \tag{10}$$

where $\theta_1 < \theta_2$, $\theta_1 \leq x \leq \theta_2$, and $\theta_1 \leq m \leq \theta_2$. The undistorted expectation is $\mu_0 = (\theta_1 + \theta_2 + m)/3$.

Finally, when the risk X is distributed $U(\theta_1, \theta_2)$, the survivor function is given by

$$S(x) = \begin{cases} 1 - \dfrac{x - \theta_1}{\theta_2 - \theta_1} & \text{if } \theta_1 \leq x \leq \theta_2 \\ 0 & \text{otherwise} \end{cases}, \tag{11}$$

and the undistorted expectation is $\mu_0 = (\theta_1 + \theta_2)/2$. Table 4 summarizes the distorted survivor function and risk measure.

It is important to note that, for the intractable results in Tables 1–3, the distorted risk measure may be approximated using numerical quadrature routines widely available in standard computing environments. In the next section, we propose measures that may be used to assess the effect of distortion and present a designed experiment to assist in establishing guidelines for appropriate distortion function selection.

MEASURING DISTORTION EFFECTS

In this section, we propose measures of effectiveness and efficiency to assist in selecting

Table 2. Distorted risk measures when $X \sim \text{Weib}(\beta, \theta)$

Distortion function	$\hat{S}(x)$	$\hat{E}[X]$
g_{PH}	$e^{a(-x/\theta)^{\beta}}$	$\frac{\theta}{\beta \sqrt[\beta]{a}} \Gamma\left(\frac{1}{\beta}\right)$
g_{DP}	$1 - (1 - e^{(-x/\theta)^{\beta}})^{b}$	$\int_0^{\infty} [1 - (1 - e^{(-x/\theta)^{\beta}})^{b}]\, dx$
g_{EX}	$\frac{1 - \exp(-e^{(-x/\theta)^{\beta}}/c)}{1 - \exp(-1/c)}$	$\int_0^{\infty} \frac{1 - \exp(-e^{(-x/\theta)^{\beta}}/c)}{1 - \exp(-1/c)}\, dx$

Table 3. Distorted risk measures when $X \sim \text{Tria}(\theta_1, \theta_2, m)$

Distortion function	$\hat{S}(x)$	$\hat{E}[X]$
g_{PH}	$\left(1 - \frac{(x-\theta_1)^2}{(\theta_2-\theta_1)(m-\theta_1)}\right)^{a}, \theta_1 \le x \le m$	$\int_{\theta_1}^{m} \left(1 - \frac{(x-\theta_1)^2}{(\theta_2-\theta_1)(m-\theta_1)}\right)^{a} dx$
	$\left(\frac{(\theta_2-x)^2}{(\theta_2-\theta_1)(\theta_2-m)}\right)^{a}, m < x \le \theta_2$	$+ \frac{(\theta_2-m)^{a+1}}{(2a+1)(\theta_2-\theta_1)^{a}}$
g_{DP}	$1 - \left(\frac{(x-\theta_1)^2}{(\theta_2-\theta_1)(m-\theta_1)}\right)^{b}, \theta_1 \le x \le m$	$m - \theta_1 - \frac{(m-\theta_1)^{b+1}}{(\theta_2-\theta_1)^{b}(2b+1)} +$
	$1 - \left(1 - \frac{(\theta_2-x)^2}{(\theta_2-\theta_1)(\theta_2-m)}\right)^{b}, m < x \le \theta_2$	$\int_{m}^{\theta_2} \left[1 - \left(1 - \frac{(\theta_2-x)^2}{(\theta_2-\theta_1)(\theta_2-m)}\right)^{b}\right] dx$
g_{EX}	$\frac{1 - \exp\left(\frac{-1}{c} + \frac{(x-\theta_1)^2}{c(\theta_2-\theta_1)(m-\theta_1)}\right)}{1 - \exp(-1/c)}, \theta_1 \le x \le m$	$\int_{\theta_1}^{m} \frac{1 - \exp\left(\frac{-1}{c} + \frac{(x-\theta_1)^2}{c(\theta_2-\theta_1)(m-\theta_1)}\right)}{1 - \exp(-1/c)} dx$
	$\frac{1 - \exp\left(\frac{-(\theta_2-x)^2}{c(\theta_2-\theta_1)(\theta_2-m)}\right)}{1 - \exp(-1/c)}, m < x \le \theta_2$	$+ \int_{m}^{\beta_2} \frac{1 - \exp\left(\frac{-(\theta_2-x)^2}{c(\theta_2-\theta_1)(\theta_2-m)}\right)}{1 - \exp(-1/c)} dx$

Table 4. Distorted risk measures when $X \sim U(\theta_1, \theta_2)$

Distortion function	$\hat{S}(x)$	$\hat{E}[X]$
g_{PH}	$\left(1 - \frac{x-\theta_1}{\theta_2-\theta_1}\right)^{a}$	$(\theta_2 - \theta_1)\left(\frac{1}{a+1}\right)$
g_{DP}	$1 - \left(\frac{x-\theta_1}{\theta_2-\theta_1}\right)^{b}$	$(\theta_2 - \theta_1)\left(\frac{b}{b+1}\right)$
g_{EX}	$\frac{1 - \exp\left(-\left(1 - \frac{x-\theta_1}{\theta_2-\theta_1}\right)/c\right)}{1 - \exp(-1/c)}$	$(\theta_2 - \theta_1)\left(\frac{1 - c + ce^{-1/c}}{1 - e^{-1/c}}\right)$

distortion functions and their associated parameters. However, we first introduce a measure of the magnitude of probability density displacement that results from the application of a distortion function. This measure of density translation uses the median of the risk distribution, namely that point at which the undistorted distribution is partitioned with equal density on either side. After distorting the original risk distribution, the magnitude of density translated from the left of the undistorted median to the right of the median using distortion function g is computed as

$$R_g \equiv \frac{(g \circ S)(\psi)}{S(\psi)}, \tag{12}$$

where $\psi \equiv \inf\{x > 0 : S(x) = 0.5\}$ denotes the median of the undistorted risk distribution, S is the undistorted survivor function, and $(g \circ S)$ is the distorted survivor function. Since the distortion functions used in this research all shift density to the right, we see that

$$1 \leq R_g \leq 2,$$

since by this ratio measurement all of the density to the left of the median can theoretically be shifted to the right of the median. However, R_g does not measure how "far" this density has been shifted—it only reflects the fact that it has been translated beyond the undistorted median. Conversely, if $R_g = 1$, this implies that no distortion has been applied whatsoever.

Effectiveness and Efficiency

The primary risk measure considered in this work is the expectation of the risk random variable. Recall that expectation has a drawback in that low-frequency risk values tend to be "dampened out" by the values with the greatest relative frequency. However, distortion functions can provide the decision maker with the ability to control expectation to predictable degrees. In choosing a distortion function for a specified risk distribution, the decision maker would like to know how *effective* each candidate distortion function/parameter combination is in inflating the expectation risk measure. After applying distortion g and computing the distorted expectation $\mu_g \equiv \hat{E}(X)$, the measures can be compared to determine which distortion has the greatest effect on that risk distribution's mean. To develop the idea further, we define the following measure.

Definition 1. The effectiveness of a distortion function is defined as the ratio,

$$K = \mu_g / \mu_0, \tag{13}$$

where μ_g is the distorted risk measure obtained by applying distortion g, and μ_0 is the undistorted risk measure.

This ratio can, for example, be used to directly compare a unique distortion function/parameter combination over different distributions, measuring that combinations's effectiveness in changing each distribution's expectation as a percentage increase. Because $\mu_g \geq \mu_0$ we see that $K \geq 1$, and whenever $K = 1$, the risk distribution is undistorted. Similarly, two different distortion function/parameter combinations applied to two dissimilar risk distributions having equal K-values are deemed to be equally effective in distorting (increasing) the expectation risk measure.

Through numerical experimentation, we have observed clear contrasts in the way different distortion function/parameter pairs shift density. As applied to a single risk distribution, one combination may require significant density shift before its K-value matches that of another pairing which has a greater effect on the distribution's tail. Prototypical examples are the PH and DP distortions. The PH distortion accumulates density in the right tail while the DP accumulates it closer to the mode, so the PH generally has a greater effect on expectation. A measure to reflect the magnitude of density shift has already been established, namely R_g. For this reason, it seems beneficial to combine the two measures K and R_g into a single measure of *efficiency.*

Definition 2. The efficiency of a distortion function g is defined as the ratio of the normalized change in the risk measure to the normalized change in density given by

$$E = K / R_g. \tag{14}$$

The measure E should not be confused with the concept of statistical efficiency related to parameter estimation. Intuitively, if a distortion function/parameter combination has a large effect on the expectation risk measure while shifting a relatively small magnitude of density, then that pairing is highly efficient when applied to the given distribution.

One might ask, "Why would a decision maker care about the magnitude of density being shifted? Why isn't the effectiveness of the distortion function/parameter combination all he or she needs to know in making a selection?"

Note that without the efficiency measure, there would be no need to distinguish between two pairings with identical effectiveness; the decision maker might conclude that one is just as good as the other, even though the underlying distribution is being changed in an entirely different manner depending on the choice. As an example, consider Figure 2 which shows an undistorted Weib(2,2) distribution along with its PH ($a = 0.2$) and DP ($b = 31$) distortions. Both of the distorted distributions have $K \approx 2.24$, but the densities are hardly similar.

Furthermore, the decision maker should care a great deal about the magnitude of shifted density required to achieve a desired increase in the resulting risk measure. More specifically, the decision maker has approximated risk distributions using the inputs of subject matters experts who presumably possess expertise the decision maker is lacking. For every unit increase of the measure R_g, the decision maker is taking an additional "step" away from the recommendations of his or her advisors (and the assumed true distributions). To illustrate this point, consider again Figure 2 in which the distortions have transformed the original Weibull risk distribution into two radically different ones. Thus, it seems likely that the decision maker would prefer one of two possible courses of action in choosing a distortion function/parameter combination:

1. Achieve the maximum increase in the expectation while affecting the original risk distribution by (no more than) a specified amount; or
2. achieve a specified increase in expectation while altering the original risk distribution as little as possible.

In either case, efficiency is the measure which provides the appropriate answer.

In order to investigate the impact of the distortion parameters on these measures, we carried out a 3^k-factorial designed experiment. For this purpose, we arbitrarily selected the following distribution parameters for each of the four distributions noted earlier: Exp(3.5), Weib(2,2), Tria(1,7,4), and $U(1,7)$. The factorial design was used to study the effects of each parameter (a, b, and c) in the gamma-beta distortion, and within this factorial design, each of the involved parameters was required to have relatively equal power over the R_g measure so that the interaction effects could be analyzed in a "fair" manner. Since we chose a face-centered cube design, three equally-spaced values were used for each parameter. Table 5 summarizes the selected distortion parameter values (or treatments). Recall that when $a = 1$, $b = 1$, and $c \to \infty$, no distortion is applied. We note that distortion is inversely proportional to the parameters a and c while it is proportional to the parameter b.

Table 6 records the efficiency and effectiveness measures for the risk distributions and single-parameter distortions studied in this paper. In general, as the amount of distortion is increased, the efficiency is decreased. There are three exceptions to the general rule, however, and the efficiency measures for these three cases are highlighted in bold face type. Specif-

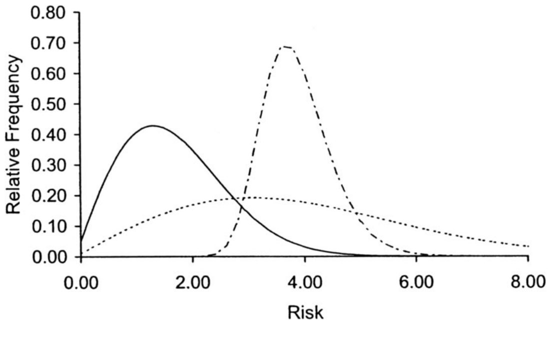


Figure 2. Distorted densities when $X \sim$ Weib(2,2) with distortion parameters $a = 0.2$ and $b = 31$ (solid is no distortion, ··· PH, – · – · DP).

Table 5. Selected distortion parameter treatments

Distortion (Parameter)	Selected Values		R_g (% density shift)
Proportional Hazard (a)	High	0.90	1.07 (7%)
	Mid	0.75	1.19 (19%)
	Low	0.60	1.32 (32%)
Dual Power (b)	Low	1.10	1.07 (7%)
	Mid	1.30	1.19 (19%)
	High	1.50	1.29 (29%)
Exponential (c)	High	3.60	1.07 (7%)
	Mid	2.20	1.11 (11%)
	Low	0.80	1.30 (30%)

Table 6. Effectiveness and efficiency measures for all distortion/distribution pairings

Distortion →	PH			DP			EX		
Measure ↓	$a = 0.9$	$a = 0.75$	$a = 0.6$	$b = 1.1$	$b = 1.3$	$b = 1.5$	$c = 3.6$	$c = 2.2$	$c = 0.8$
Exp(3.5), $\mu_0 = 0.285714$									
μ_g	0.3175	0.3810	0.4762	0.3036	0.3363	0.3658	0.3058	0.3189	0.3791
R_g	1.0718	1.1892	1.3193	1.0670	1.1877	1.2929	1.0693	1.1131	1.3027
K	1.1111	1.3333	1.6667	1.0625	1.1772	1.2803	1.0704	1.1161	1.3270
K/R_g	**1.0367**	**1.1212**	**1.2633**	0.9958	0.9911	0.9903	**1.0010**	**1.0027**	**1.0186**
Weib(2,2), $\mu_0 = 1.772454$									
μ_g	1.8683	2.0467	2.2882	1.8448	1.9713	2.0788	1.8449	1.8911	2.0971
R_g	1.0719	1.1895	1.3199	1.0670	1.1879	1.2932	1.0694	1.1133	1.3032
K	1.0541	1.1547	1.2910	1.0408	1.1122	1.1729	1.0408	1.0669	1.1831
K/R_g	**0.9834**	**0.9707**	**0.9781**	0.9755	0.9362	0.9069	0.9733	0.9584	0.9079
Tria(1,7,4), $\mu_0 = 4.000$									
μ_g	4.1163	4.3218	4.5777	4.1033	4.2793	4.4246	4.0971	4.1586	4.4275
R_g	1.0718	1.1892	1.3193	1.0670	1.1877	1.2929	1.0693	1.1131	1.3027
K	1.0291	1.0804	1.1444	1.0258	1.0698	1.1062	1.0243	1.0396	1.1069
K/R_g	0.9602	0.9086	0.8675	0.9614	0.9007	0.8556	0.9579	0.9340	0.8497
$U(1,7)$, $\mu_0 = 4.000$									
μ_g	4.1579	4.4285	4.7500	4.1428	4.3913	4.6000	4.1387	4.2265	4.6093
R_g	1.0718	1.1892	1.3193	1.0670	1.1877	1.2929	1.0693	1.1131	1.3027
K	1.0395	1.1071	1.1875	1.0357	1.0978	1.1500	1.0347	1.0566	1.1523
K/R_g	0.9699	0.9310	0.9001	0.9707	0.9243	0.8895	0.9676	0.9492	0.8846

ically, in the case of the exponential distribution, efficiency increases with distortion when using the PH and EX distortion functions. For the PH distortion applied to the Weibull distribution, efficiency at first decreases as distortion is increased, then begins to increase again. A brief investigation to verify this result showed that the least efficiency occurs at about $a = 0.72$.

Using Table 6, some general rules (within the limits of this study) can be established for selecting a distortion function to apply to a distribution. Recall that a decision maker would likely be interested in either (i) achieving the largest possible increase in the mean given a specified maximum shift in density, or (ii) shifting the density by the smallest amount required to achieve a specified increase in expectation. Using Table 6, some answers may be available when objective (i) is of primary importance. Table 7 was created from Table 6 by comparing efficiency across categorized values of R_g. For example, considering the triangular distribution in Table 6, the low-distortion efficiency values are 0.9602 for the PH ($a = 0.9$), 0.9614 for the DP ($b = 1.1$), and 0.9579 for the EX ($c = 3.6$). Since the DP value is the highest, this was entered into the appropriate cell of Table 7. Thus in the case of objective (i) when assuming a triangular risk distribution, the DP distortion is the most efficient (although the values are relatively close in this case).

In examining Table 7, note once again that the difference in the R_g values between the PH and DP distortions ($R_g \approx 1.19$) and the EX distortion ($R_g \approx 1.11$) at the "moderate" distortion level could be significant in the final selection of a distortion function at that level. In addition, note that decision maker objective (ii)

Table 7. Suggested distortions for selected distributions (via efficiency)

Risk Distribution	Low Distortion (0–10%)	Moderate Distortion (11–20%)	Heavy Distortion (21–30%)
Exp(3.5)	PH	PH	PH
Weibull(2,2)	PH	PH	PH
Tria(1,7,4)	DP	EX	PH
$U(1,7)$	DP	EX	PH

could be answered just as easily as objective (i), but the original response surface study which facilitated the distortion parameter choices would have had to fix the distorted *expectations* rather than the *amount of density shift* being applied.

Some Distortion Selection Guidelines

In this subsection, we summarize some conclusions that can be drawn from our simple designed experiment regarding the selection of distortion functions and their associated parameters. It is important to note that these results are not generalizable to all risk scenarios, but illustrate the means by which one might make such selections for the distributions we have considered here. More specifically, our conclusions are valid especially when the highly adverse outcomes correspond to the right tail of the distribution (i.e., when the decision maker desires to shift probability density to the right).

1. When an exponential or Weibull distribution is appropriate for the risk scenario, the PH distortion is drastically more efficient than the DP or EX. In the case of the exponential distribution, the PH also leaves the mode in place at zero, while other distortions "pull" the mode away from zero.
2. For the triangular and uniform distributions, no distortion appears to be as totally dominant (in efficiency) as the PH is for the exponential and Weibull. For each of these bounded distributions, the DP distortion is the most efficient in cases where only a small amount of distortion is required; questionably, the EX is more efficient in the vicinity of $R_g = 1.15$; and the PH is most efficient when larger amounts of distortion are required.
3. If higher moments are desired from the distorted distribution (e.g., the variance may well be of concern), then the DP and EX distortions may be preferred over the PH. Particularly in the case of the Weibull and triangular distributions, the DP accumulates density near the mean, likely reducing the impact on variance.
4. The parameter b of the DP distortion has a meaningful interpretation. In particular, it corresponds to the expected value of the worst outcome when b samples are taken from the random variable (Wirch and Hardy 1999). If the decision maker appreciates this interpretability but wishes to use either the PH or EX distortion, a value of b can be obtained which results in a DP match in μ_g to the specified a or c parameter. In this manner, the interpretability can be "loaned" to the PH and EX distortions through a single extra step.

In the next section, we illustrate the means by which distorted risk measures may be employed to incorporate the risk of capability shortfalls in a resource allocation problem.

RESOURCE ALLOCATION AND DISTORTION: AN ILLUSTRATION

Suppose there are nine distinct areas of military (specifically fighter aircraft) capability that might be of interest to a decision maker. Table 8 provides the descriptions of such notional areas.

A shortfall in capability area i creates a risk (say X_i) with undistorted risk measure $E(X_i)$ and distorted risk measure $\hat{E}(X_i)$, $i = 1, 2, \ldots, 9$. However, to address shortfalls in capability (i.e., to mitigate risk), six distinct risk-mitigating systems may be acquired as summarized in Table 9.

Let $m_{i,j}$ denote the shortfall mitigation to area i ($i = 1, 2, \ldots, 9$) obtained from acquisition of system j ($j = 1, 2, \ldots, 6$). For instance, if $m_{i,j} = 0.50$, then the purchase of system j reduces the shortfall in capability area i by 50%. We initially assume that the decision maker can choose to acquire some or all of a risk-mitigating system. Let x_j represent the proportion of system j to be acquired so that $0 \leq x_j \leq 1$, $j = 1, 2, \ldots, 6$ (i.e., partial investments are permissible without loss of contribution). Next, define c_j as the cost of acquiring one complete unit of system j. The decision maker's objective is to maximize the risk mitigation by strategically choosing the proportion of various systems to purchase, subject to a fixed budget B. The op-

Table 8. Notional aircraft capability areas and descriptions

Area (i)	Name	Description
1	Reconnaissance	Locate specific areas and record information about those areas
2	Range and Payload	Combat radius, weapon types and quantities
3	Communications	Ability to transmit and receive messages
4	Passive Sensors	Detection ability without broadcasting a radio frequency (RF) signal
5	Offensive Firepower	Ability to employ armament against ground and airborne targets
6	Self-Defense	Ability to defend against infrared- and RF-guided threats
7	Life Support	Ability to protect the pilot and provide for human needs (e.g., oxygen)
8	Networking	Ability of the aircraft to integrate into the battle space
9	Availability	The proportion of time the aircraft is available for its missions

Table 9. Potential risk-mitigating systems

System (j)	Description
1	Color cockpit display
2	Enhanced mission computer
3	New air-to-ground weapons system
4	New helmet-mounted targeting system
5	Enhanced digital radar system
6	Improved ground support equipment

timal strategy may be determined by solving the linear program (LP),

$$\max \sum_{i=1}^{9} \sum_{j=1}^{6} \hat{E}(X_i) m_{i,j} x_j \tag{15a}$$

$$\text{s.t.} \sum_{j=1}^{6} c_j x_j \leq B \tag{15b}$$

$$0 \leq x_j \leq 1,\ j = 1,2, \ldots, 6. \tag{15c}$$

An intuitive explanation of the LP is as follows. Our decision maker would like to selectively apply distortion to the various risk distributions in order to better reflect his or her own risk priorities. For this reason, in (15a), we weight each $m_{i,j}$ by the distorted expectation risk measure, $\hat{E}(X_i)$, $i = 1, 2, \ldots, 9$. If we instead use the undistorted values, $E(X_i)$, $i = 1, 2, \ldots, 9$, this corresponds to solving the problem using only the information provided by subject matter experts based solely on the unadjusted expected risk (i.e., ignoring the decision maker's concerns about catastrophic loss). If we set $\hat{E}(X_i) = 1$ for each i in (15a), this is equivalent to ignoring the risks altogether.

With regard to the budgetary constraint (15b), we have assumed in this example that the cost of acquiring system j is linear in the proportion of investment in system j. Of course, it is possible that the costs may differ if funding of only part of a risk-mitigating system is selected. A number of approaches in the literature are available to model various operational settings if other conditions apply (see Martello and Toth 1990 among others). Nonetheless, we solve the LP under this assumption for the purpose of illustrating the methodology. It is worth noting that Woodward (2004), used an integer programming approach and a distorted expectation risk measure, but uniformly applied the DP distortion function with a constant parameter b across all capability areas. Our approach permits the decision maker to vary the type and level of distortion in each area while also allowing for the possibility of partial investments when appropriate.

We now illustrate solving the LP in a specific problem instance. We assume the budget is fixed at $B = 25$ monetary units. Suppose the decision maker is least risk averse to catastrophic loss in capability areas 8 and 9, somewhat risk averse to catastrophic loss in areas 4, 6, and 7, and most risk averse to catastrophic loss in areas 1, 2, 3, and 5. The weights summarized in Table 10 reflect the decision maker's degree of risk aversion in each area. In particular, a higher weight represents greater risk aversion. The nine distributions, assumed to originate from the inputs of nine teams of subject matter experts, are also included. Specifi-

Table 10. Notional data for illustrative example

Area (i)	Weight	Distribution	$\mu_0 = E(X_i)$
1	20	Weib(3.5,3.3)	2.9692
2	30	Tria(0,4.67,3.2)	2.6233
3	19	$U(0,4)$	2.0000
4	13	Tria(0,4,2)	2.0000
5	46	Weib(2.04,1.74)	1.5416
6	6	Weib(3.08,2.84)	2.5391
7	6	$U(1,3)$	2.0000
8	0	Exp(0.45)	2.2222
9	0	Tria(0,1.875,0.5)	0.7917

cally, these distributions correspond to the risk associated with shortfalls in the respective capability areas summarized in Table 8.

We assume the decision maker would like to impose one of two priorities: (i) obtain the greatest possible increase in the expectation risk measure given a specified shift in density, or (ii) minimize the magnitude of density shifted to achieve a specified increase in the expectation. While either priority may be considered, we will proceed with this example on the assumption that the decision maker prefers objective (i), and that the degree of risk aversion (i.e, the weights in Table 10) assigned to an area corresponds to a specific shift in density R_g. For instance, for Area 1 (reconnaissance capability), the risk distribution is assumed to be Weib(3.5,3.3), and the decision maker has chosen, based on available information and projections, to shift 20% of the density beyond the median, or $R_g \approx 1.20$. At this level, using this distribution, the PH distortion is the most efficient. Setting $a = 0.735$ results in a distorted expectation of $\mu_g = 3.2422$. We continue in this fashion for all risk distributions, applying distortions based on the recommendations of Table 7. Table 11 summarizes the results of selectively distorting as per the prespecified preferences of the decision maker. The column entitled, "Distortion" is the selected distortion function and its associated parameter value.

Table 12 summarizes the impact of each system on mitigating capability shortfalls. The table elements correspond to the percent shortfall mitigation that each potential acquisition addresses in all nine areas. For example, system 2 mitigates the risk of a shortfall in area 1 by 19%.

The last row of Table 12 is the cost associated with the purchase of a *complete* system.

Assuming a budget of 25 units, Table 13 summarizes the optimal solution to the resource allocation problem when formulated as an LP. The "Unweighted" solution assumes that we do not weight the mitigation terms by a risk measure at all. The rows entitled, "Weighted, Undistorted" and "Weighted, Distorted" provide the solutions using undistorted and distorted risk measures, respectively. In Table 13, an entry of 1.0 represents a recommendation to purchase a complete system, a decimal represents a partial purchase, and 0.0 represents no purchase.

The different optimal solutions for the three scenarios agree with intuition. In particular, we note that systems 1 and 2 are consistently chosen because they significantly mitigate the risk of shortfalls in capability areas 2–5 which the decision maker has deemed to be

Table 11. Selection of distortion functions and parameters

Area (i)	Distribution	R_g	Distortion	$\mu_g = \hat{E}(X_i)$
1	Weib(3.5,3.3)	1.20	PH, $a = 0.735$	3.2422
2	Tria(0,4.67,3.2)	1.30	PH, $a = 0.062$	3.0197
3	$U(0,4)$	1.19	EX, $c = 1.300$	2.2539
4	Tria(0,4,2)	1.13	EX, $c = 1.900$	2.1223
5	Weib(2.04,1.74)	1.46	PH, $a = 0.450$	2.2801
6	Weib(3.08,2.84)	1.06	PH, $a = 0.915$	2.6134
7	$U(1,3)$	1.06	DP, $b = 1.090$	2.0431
8	Exp(0.45)	1.00	N/A	2.2222
9	Tria(0,1.875,0.5)	1.00	N/A	0.7917

Table 12. Percent shortfall mitigation

Area (i)	$m_{i,1}$	$m_{i,2}$	$m_{i,3}$	$m_{i,4}$	$m_{i,5}$	$m_{i,6}$
1	0.00	0.19	0.00	0.26	0.26	0.00
2	0.46	0.21	0.00	0.12	0.00	0.00
3	0.34	0.19	0.00	0.05	0.00	0.23
4	0.14	0.42	0.00	0.36	0.00	0.05
5	0.10	0.21	0.92	0.30	0.00	0.10
6	0.00	0.00	0.54	0.00	0.00	0.11
7	0.16	0.00	0.00	0.00	0.25	0.00
8	0.19	0.00	0.00	0.00	0.31	0.48
9	0.00	0.00	0.00	0.00	0.33	0.36
c_j	7.0	7.0	10.0	8.0	8.0	9.0

Table 13. Optimal solutions under various scenarios (LP formulation)

Weighting Scheme	x_1	x_2	x_3	x_4	x_5	x_6
Unweighted	1.000	1.000	0.200	0.000	0.000	1.000
Weighted, Undistorted	1.000	1.000	0.300	1.000	0.000	0.000
Weighted, Distorted	1.000	1.000	1.000	0.125	0.000	0.000

relatively important areas. On the contrary, systems 5 and 6 are seldom chosen due to the fact that they do not impact the critical areas (2–5) in a significant way. System 3 becomes an important asset once the decision maker's preferences are included because it has the potential to mitigate risks in a shortfall of capability area 5, the area in which the decision maker is most risk averse.

For the sake of comparison, we next consider a binary integer programming (BIP) formulation of the problem where constraint (15c) is replaced by the constraint $x_j \in \{0, 1\}$, $j = 1, 2, \ldots, 6$. Table 14 summarizes the optimal solutions in this case.

As expected, when partial investments are prohibited, the optimal solutions differ (see for example Taha 1975). However, we note that in this illustrative binary IP formulation, systems 5 and 6 remain unimportant in mitigating the risk of capability shortfalls while system 1 is *always* selected when the risk measures are included. When choosing between the LP, BIP or a mixed model, of course, the analyst will select the model that most accurately represents the decision environment. In the conclusion, we provide some final remarks and possible directions for future inquiry.

CONCLUSIONS

The properties of distortion functions have been well documented in the current risk analysis literature. However, the appropriate selection of a distortion function and its corresponding parameters is a problem that has not

Table 14. Optimal solutions under various scenarios (BIP formulation)

Weighting Scheme	x_1	x_2	x_3	x_4	x_5	x_6
Unweighted	1.000	0.000	1.000	0.000	1.000	0.000
Weighted, Undistorted	1.000	0.000	1.000	1.000	0.000	0.000
Weighted, Distorted	1.000	0.000	1.000	1.000	0.000	0.000

received much attention. This study takes an initial step toward addressing this important issue. Our primary objective was to provide some practical recommendations for risk analysts who seek to use distortion functions to adjust the expectation risk measure to better account for low-likelihood yet potentially catastrophic events. For our purposes, we considered risks that may arise from shortfalls in military or homeland security capabilities which may result in obvious detrimental outcomes.

We have provided a procedure, via analytical and empirical methods, for the selection of distortion functions and their parameters on a set of parametric risk distributions. The use of distortion functions provides a tractable and documentable procedure to investigate the shifting of risk in the face of catastrophic events. Two new measures, efficiency and effectiveness, were proposed to distinguish the effects of different distortions and to make basic recommendations regarding the appropriateness of certain distortion functions and parameters using specific risk distributions. Additionally, a linear programming model was formulated to illustrate the means by which the distorted expectation risk measure can be used to influence the acquisitions plan of a risk-averse decision maker.

There are some obvious shortcomings in this work which are noted here. First, the selection guidelines that we provide are limited to the risk distributions considered in this study. Of course, it will be important to consider a wider range of distributions and to study the interaction between distribution and distortion function parameters. Moreover, we considered only the expectation risk measure, and it may prove useful to consider other coherent measures in the future. Another limitation stems from the use of the quantity R_g which measures only the magnitude of density shifted beyond the median of the undistorted risk distribution. This measure really does not tell us how "far" beyond the median the density has been translated. Other measures should be considered, as should a more comprehensive risk measure such as those described in Sarin and Weber (1993). It may also be instructive to investigate the relationship between the skewness of the risk distribution (perhaps using Pearson's skewness coefficient) and either the percent change in expectation or R_g. There appears to exist some correlation between Pearson's coefficient and the normalized mean. That is, the mean of some risk distributions (the exponential is one case) is more sensitive to the application of distortion than others. Finally, further research regarding the effects of distortion on variance may significantly impact the selection of distortion functions for specific risk scenarios.

ACKNOWLEDGMENTS

The authors thank two anonymous referees for their insightful comments and suggestions. We also wish to thank Dr. Edward Pohl who acted as Editor for this paper.

REFERENCES

Artzner, P., Delbaen, F., Eber, J.-M., and D. Heath (1997). Thinking coherently. *Risk*, 10: 68–71.

Artzner, P., Delbaen, F., Eber, J.-M., and D. Heath (1999). Coherent measures of risk. *Mathematical Finance*, 9: 203–228.

Benati, S. (2003). The optimal portfolio problem with coherent risk measure constraints. *European Journal of Operational Research*, 150: 572–584.

Clemen, R.T. and T. Reilly (2001). *Making Hard Decisions with Decision Tools Suite*, 2nd Edition. Pacific Grove, CA: Duxbury.

Lambert, J.H., Matalas, N.C., Ling, C., Haimes, Y.Y., and D. Li (1994). Selection of probability distributions in characterizing risk of extreme events. *Risk Analysis*, 14 (5): 731–742.

Martello, S. and P. Toth (1990). *Knapsack Problems: Algorithms and Computer Implementations*, John Wiley & Sons, NY.

McLeish, D.L., and R.M. Reesor (2003). Risk, entropy, and the transformation of distributions. *North American Actuarial Journal*, 7: 128–144.

Sarin, R.K., and M. Weber (1993). Risk-value models. *European Journal of Operational Research*, 70: 135–149.

Taha, H.A. (1975). *Integer Programming: Theory, Applications, and Computations*, Academic Press.

Wang, S. (1995). Insurance pricing and increased limits ratemaking by proportional hazards transforms. *Insurance: Mathematics and Economics*, 17: 43–54.

Wang, S. (1996a). Premium calculation by transforming the layer premium density. *ASTIN Bulletin*, 26: 71–92.

Wang, S. (1996b). Ordering of risks under PH-transforms. *Insurance: Mathematics and Economics*, 18: 109–114.

Wang, S.S., Young, V.R., and H.H. Panjer (1997). Axiomatic characterization of insurance prices. *Insurance: Mathematics and Economics*, 21: 173–183.

Wirch, J.L., and M.R. Hardy (1999). A synthesis of risk measures for capital adequacy, *Insurance: Mathematics and Economics*, 5: 337–347.

Woodward, W.E. (2004). *Measuring the Risk of Shortfalls in Air Force Capabilities.* M.S. Thesis, Department of Operational Sciences, Air Force Institute of Technology, Dayton, OH.

MORS
Military Operations Research Society

2006 Membership Application

MAIL OR FAX TO: Military Operations Research Society, (MORS) Suite 450, 1703 N. Beauregard Street, Alexandria, VA 22311-1745
703-933-9066 - Phone: 703-733-9070 - E-mail: morsoffice@mors.org

******NEW* in 2006 – Join MORS*****

Name:	Date:

Mailing address preference: ☐ **Home** ☐ **Work**	Organization (optional):
Home address:	Address:
City – State – Zip (COUNTRY)	City – State – Zip (COUNTRY)

Phone:	Fax:	Email:

United States **Membership Dues*** **(A subscription to PHALANX is included in the membership dues)**	☐ **$ 75 (1 year)**	☐ **$140 (2 years)**	☐ **$210 (3 years)**
International **Membership Dues*** **(A subscription to PHALANX is included in the membership dues)**	☐ **$ 100 (1 year)**	☐ **$190 (2 years)**	☐ **$285 (3 years)**

* **Benefits of MORS Membership!** 1 year membership dues includes: A subscription to ***PHALANX***; Discount on MORS Meeting Fees during the membership term (*Membership does not guarantee invitations to all MORS meetings*); MORS Lapel Pin; and, Membership Card. Multiple-year memberships are available and include a subscription to *PHALANX* for all membership years.

MOR **JOURNAL** ***United States*** **Subscription Rates:**	☐ **$ 70 (1 year)**	☐ **$130 (2 years)**
MOR **JOURNAL** ***International*** **Subscription Rates:**	☐ **$ 135 (1 year)**	☐ **$260 (2 years)**

☐ **CASH** ☐ **CHECK** ☐ **VISA** ☐ **MC** ☐ **AMEX**	**TOTAL** $

Credit Card #:	**Expiration date:**
Name on Credit Card (print):	**Phone:**
Billing address and zip code (COUNTRY):	**Zip Code:**

Signature:	**Date:**	**Sold by:**	**Date shipped/picked-up:**

Revised 7/11/06

Estimating Total Program Cost of a Long-Term, High-Technology, High-Risk Project with Task Durations and Costs That May Increase Over Time

Dr. Gerald G. Brown

Naval Postgraduate School
gbrown@nps.edu

Maj Roger T. Grose

Australian Army

Dr. Robert A. Koyak

Naval Postgraduate School
rakoyak@nps.navy.mil

APPLICATION AREAS:
Analysis of alternatives and cost analysis
OR METHODOLOGIES:
Linear programming, simulation

ABSTRACT

We plan a long-term project schedule for which the total budget depends upon the year the project finishes. Each task in the project can begin only when all its predecessor tasks have been completed, and each task has a range of feasible durations with a month-by-month cost profile for each duration. A task start can be delayed, but once started for some chosen duration, a task cannot be interrupted. Each task suffers some risk of delay and changed cost. Ignoring budget constraints, we use Monte Carlo simulation of the duration of each task in the project to infer the probability distribution of the project completion time. We then optimize a deterministic project schedule following budget guidance. Finally, we successively reschedule as the project progresses, simulating annual review of active tasks, and possibly delaying each active task's duration and changing its monthly costs for its forecast duration. We do not require an independence assumption, so we can accommodate learning effects from completed tasks. U.S. Army Future Combat Systems (FCS) is our motivating application. FCS is a complex of information technologies, sensors, and command systems expected to require more than a decade and $16 billion to develop. The U.S. General Accounting Office finds FCS at significant risk of cost and schedule growth, and suggests two alternatives to a baseline Army plan. We analyze these three alternate project plans for FCS to discover which one can most likely be completed soonest and cheapest.

> *"Now, I'll manage better this time."* ***Alice in Wonderland***

INTRODUCTION

U.S. Army Future Combat Systems (FCS) is a complex of information technologies, sensors, and command systems constituting a project with scores of tasks expected to require more than a decade and $16 billion (2004 U.S. dollars) just in system development and demonstration costs. In fiscal year (FY) 2005, FCS is expected to consume more than half of the U.S. Army's budget for all system development and demonstration, and perhaps $94 billion to acquire 14 of the 18 systems needed for FCS initial operational capability by the year 2010 (Brady, 2003; Francis, 2004). The U.S. General Accounting Office (GAO) (2003) finds FCS vulnerable to significant cost and schedule growth, and suggests alternate project designs to mitigate risk.

Francis (2004) outlines the accomplishments that must be coordinated in order for FCS to succeed, which we paraphrase:

- A specialized C4ISR (Command, Control, Communications, Computer, Intelligence, Surveillance, and Reconnaissance) network must be developed for FCS;
- Fourteen major weapon systems and platforms must be designed and integrated simultaneously with other systems, subject to physical limitations;
- At least 53 technologies that are considered critical to achieving required performance capabilities must be matured and integrated;
- At least 157 Army and joint-forces systems must also be adapted to interoperate with FCS, which will require the development of nearly a hundred new network interfaces; and
- An estimated 34 million lines of software code will be required to operate FCS. This is nearly five times the software required for the Joint Strike Fighter, which had the largest software requirement of any Department of Defense acquisition prior to FCS.

FCS is so complex, a number of normal procedural reviews and hurdles have been relaxed, enabling an independent initial operational test and evaluation using an incomplete prototype scheduled for 2008 (Welch, 2003).

We seek a "project design" for such a long-term, high-risk, complex system. We anticipate that higher-risk tasks will exhibit more uncertainty and thus may take longer than planned and cost more. We are willing to state probability distributions predicting the cost and duration of each task, but we view an independence assumption between task outcomes as foolhardy: In complex, high-technology projects, trouble breeds company.

We seek a "robust project schedule" that offers the least schedule risk. We want to plan to complete our project at some given budget, by some given time, with some given probability. We can rearrange some of the planned project *partial orders* among tasks—i.e., what predecessor tasks have to be completed before any given task can start—and these rearrangements might influence schedule robustness. Our problem is: which rearrangement offers the most robust project schedule?

SCHEDULE OPTIONS AND SCHEDULE RISK

We refer to *schedule risk* as the costs of schedule overruns evaluated by their likelihoods. Planners might be presented with a set of options for scheduling the range of tasks that comprise a large acquisition project. These options must abide by a common set of temporal and fiscal constraints. They should also reflect the inherent uncertainty of the completion time of a developmental task. A rational planner assesses the schedule risk of each option and selects the option that affordably poses the least risk.

Significant "knowledge demonstration" (i.e., showing you can actually build components that integrate in the system design) often occurs late in development and early in production of a major defense acquisition program. The highest schedule risk comes when developed components must be integrated into a system of systems. Welch (2003) observes that the unusual complexity of FCS exposes it to higher schedule integration risk than normally expected of a major program. In particular, FCS is susceptible to "late cycle churn" to fix problems discovered late in development. Francis (2004) identifies the following factors that dispose FCS to late cycle churn, which again we paraphrase:

- Technology development is expected to continue through to the production decision;
- Technology development will still be ongoing at the design readiness review, putting at risk the stability of ongoing system integration;
- Production is planned to start while technology development and system integration are continuing and the first prototypes are being delivered;
- The final production decision will be made before some technologies reach their required maturation and before an integrated system demonstration has been conducted;
- Production delivery will start before the Army has completed the first full demonstration of FCS as an integrated system; and
- The full-rate production decision will be made while testing and demonstration are continuing.

The FCS program executive office has prepared a baseline project plan (i.e., a schedule with funding) for the system development and demonstration phase that governs current acquisition policy. Several alternate project plans have been proposed by the General Accounting Office (2003) to mitigate FCS schedule risks. We examine the baseline plan and two of the GAO alternatives here.

1. "FCS baseline" plan

The baseline plan develops all major subsystems concurrently, rather than developing one first to set the development context for follow-on systems. The FCS program executive office acknowledges that this plan is ambitious, and that the program was not ready for system development and demonstration when it was approved (Francis, 2004).

2. "GAO risk first" plan

This plan modifies the baseline to address risky technologies up front, requiring that the technology readiness level (a gauge of completion) be "at least 6" to pass intermediate review and "at least 7" to qualify for production (Wynne, 2003). Many key technologies are below the 6 threshold, and the FCS program executive office has already developed risk-mitigation strategies for each. This GAO suggestion first matures technologies that are below the technical readiness-6 threshold, and then proceeds as scheduled in the baseline plan. The advantage is that test and integration tasks occur later in the schedule, with theoretically reduced schedule risk compared to the baseline plan.

3. "GAO C4ISR first" plan

This plan modifies the baseline to develop C4ISR tasks before all others. The C4ISR components are believed to pose the greatest schedule risks to FCS development due to their scope and complexity. They are expected to require about 16 million lines of software code (of the 34 million total estimated), of which more than half will be new code (Welch, 2003). This huge undertaking is vulnerable to cost and schedule overruns. By investing early in these components, subsequent C4ISR test and integration tasks should pose less risk than in the baseline.

Key distinctions between these alternate plans are that the partial orders among tasks may change between plans and any task common to all of the plans may be allocated different risk levels in each. For instance, "system integration and testing" is high-risk in the "baseline plan" because immature technologies must be concurrently developed and integrated, but this same task has lower risk in the "GAO risk first" plan.

The three alternate plans displayed in the Appendix use nominal (i.e., unclassified, nonproprietary) FCS task data provided by the Cost Analysis Improvement Group, Program Analysis and Evaluation (PAE), Office of the Secretary of Defense.

TIME FIDELITY

Monthly time fidelity suffices for purposes of long-term planning and budgeting, although it is also customary to offer annual budget accounting for such plans and perhaps to conduct annual reviews of task progress. Indeed, annual task reviews are the most substantive control points in such projects, given that they are tied to annual budget authorizations. Accordingly, we plan all activities and events in months, but make annual task-state reviews with possible consequences on task duration and time.

EVALUATE EACH "PROJECT DESIGN" FOUR WAYS

We were asked to analyze the FCS baseline plan and the two GAO alternatives. The following is essentially a series of project reports as we went back to our PAE sponsor with intermediate results, seeking guidance for the next steps to try. We do not recount a lot of ideas that did not work. Overall, we spent 8 person-weeks with PAE, and 24 person-weeks finding out what works, and what doesn't. Remember: *the goal here is discovering new, effective ways to improve cost estimation for this huge, complex project, not manage it.*

First, we just find the deterministic project duration (i.e., the "shortest longest path length" in time, or simply the "critical path length."). This is easy, and exercises our newly-completed scenario data sets. We are still debugging and scrubbing data.

Then, we ignore costs and budgets, but assert probability distributions for task durations and apply Monte Carlo simulation to evaluate the critical path induced from each sampled project instance. The statistics we gather, and experience we gain, helps us understand the behavior of each project design, especially the partial orders among tasks.

Next, we provide a list of total project durations in years and a total program budget for achieving each of these durations. We specify the year-by-year spending goal of any selected project duration. Each task can be started only when all its predecessors in the project design have been completed. Each task can be started for any of a range of durations in months, and each of these durations has a monthly cost profile. Once a task is started, it cannot be interrupted. However, a task start can be delayed for lack of available budget(s) sufficient to support its chosen, uninterrupted duration once started. We optimize this deterministic, cost-constrained project schedule to minimize total project duration.

We note that "costs" need not be strictly expressed in constant-dollar allocations, but can include policy penalties rewarding desirable outcomes (i.e., finishing earlier), or penalizing bad ones (i.e., finishing very late). But, although completion time is a concern, the over-arching constraint will be total obligation authority (i.e., money) committed to the program.

Finally, we nest our cost-constrained project schedule optimization within an annual state review simulation of each *active task* (i.e., task in progress at time of review). At each annual review, each active task may be delayed depending on a probability distribution that depends on the risk of that task, *or on any prior experience with any other task.* So, year-by-year, we conduct an annual state review of all the active tasks, then reoptimize the remaining planning horizon. This takes a lot of computation, but the insights are worth the effort.

RELATED RESEARCH

Malcolm, Roseboom, Clark, and Fazar (1959) introduce Program Evaluation and Review Technique and Critical Path Method (PERT-CPM) developed for the Polaris fleet ballistic missile program, and Kelly (1961, Kelly 1963) provides a mathematical foundation. Wiest (1964) highlights two key shortcomings in CPM at its nascent stage: it only considers constant task durations and does not recognize resource constraints.

More recent concepts of CPM allow for greater flexibility in these areas, for example by allowing tasks to be scheduled in either "regular time" (with nominal costs) or in "crash time" (with higher costs), and by allowing cost constraints. Even with these innovations the concept of a "task" remains unitary in nature. At a fixed point in time of the project, tasks that are underway are not subject to decisions that affect their remaining times until completion.

If each task duration is random, and some deterministic equivalent time is used in CPM, estimates of project duration are generally optimistic as Fulkerson (1962) demonstrates using discrete random task durations. A task not on a critical path using mean durations may be on the critical path with positive probability when its duration is treated as a random variable. Dodin (1984) reports upper and lower bounds on project duration when task durations are independent random variables, and uses the Central Limit Theorem to justify treating the project duration as approximately normally distributed. While this assumption offers tractability, the longest random-length path is neither normally distributed in theory, nor in practice (as can be verified by simple Monte Carlo simulation), and this assumption can give misleading results.

Resource constraints are admitted by Bowman (1958), who introduces linear programming for CPM, and Senju and Toyoda (1968) and Pritsker, Watters, and Wolfe (1969) state integer-linear programs representing discrete decisions. Demeulemeester and Herroelen (2002) present formulations of resource-constrained project scheduling problems and review solution methods.

Using linear and integer linear programs to represent stochastic models has a long history. Babbar, Tintner, and Heady (1955), Tintner (1955, Tintner 1960), and Sengupta, Tintner, and Morrison (1963) show how to embed optimization within Monte Carlo simulation. Task duration may be treated as a random variable with a distribution not completely known (Herroelen, Reyck, and Demeulemeester, 1998). Factors influencing these random variables include resource availability, scheduling of deliveries, modification of due dates, and changes in project scope that might imply the cancellation or addition of future tasks (Herroelen and Leus, 2004).

Generally, the increased realism of stochastic PERT-CPM modeling comes at the price of increased analytic abstraction and computational cost. Deterministic equivalent objectives, such as the *expected* project critical path length or expected costs that include penalties for violating constraints (Gutjahr, Stauss, and Wagner, 2000), may be easy enough to state and solve, but the risk of such solutions is much more difficult to gauge, even given generous independence assumptions.

If task duration is random and not independent of other task durations, the distribution of the total project duration is difficult to characterize (Yang, Geunes, and O'Brien, 2001). An independence assumption is often made to render tractable analysis, but this assumption is not realistic. An optimal deterministic schedule typically has insufficient slack to remain optimal (or even feasible) in an uncertain setting, and thus lacks robustness (Herroelen, 2004). A trivial example with two identical, parallel tasks, each with random duration, reveals this property.

In addition, managers want the flexibility to change their scheduling decisions as the project evolves. *Full dynamic scheduling* offers decision points at task completions (Igelmund and Radermacher, 1983).

We need resource (essentially budget) constraints and we cannot ignore uncertainty. Of all these historical contributions, we admire Tintner's works most for their originality, elegance, and simplicity, and we follow his advice: for the stochastic modeling, use Monte Carlo identity simulation, and then use optimization for each random realization.

Finally, we do assume, as does PERT-CPM, that each task is separable and distinct from all others. In our case, these tasks are subcontracts, so this is true in law as well as in fact: If you want to re-define tasks, you must re-negotiate contracts.

FIND SHORTEST PROJECT COMPLETION DATE FOR EACH ALTERNATE PLAN WITH NO BUDGET CONSTRAINT

To check our alternate project plans to see if we get schedules that make sense, we ignore budget constraints and just solve a deterministic CPM problem.

Given a project network with fixed task durations, we wrote a Java (Sun Microsystems, 2005) procedure for an unconstrained reaching algorithm to search the project tasks over their adjacencies in partial order to find the completion time of the project. The completion time is the length of a longest path from project start to finish. This is one of the simplest network algorithms (e.g., see topological sorting and reaching in Ahuja et al., 1993, pp. 107–108), with worst-case runtime linear in the number of partial orders.

From a project start in January 2003, this primitive deterministic analysis yields an earliest project completion date of October 2012, for the "FCS baseline plan." The Army wants to field its first unit in September 2012, so this is reassuring.

Given that we can solve each of these deterministic problems in less than a millisecond, we suggested solving thousands of these problems in a Monte Carlo simulation to assess stochastic elements of each project alternative. PAE agreed.

MONTE-CARLO SIMULATE TASK DURATIONS FOR EACH ALTERNATE PLAN WITH NO BUDGET CONSTRAINT

The three-parameter Weibull distribution is often used to model the duration of developmental tasks for cost estimation and planning. Law and Kelton (2000, p. 376) explain the reasoning for the use of this distribution. The Weibull reliability function:

$$R(x;\, \alpha,\, \beta,\, \gamma) = e^{-\left(\frac{x-\gamma}{\alpha}\right)^{\beta}},\, x \geq \gamma$$

is completely characterized by its three non-negative parameters. An absolute minimum task duration is given by γ. For $\beta > 1$, the Weibull density has a mode strictly greater than γ, and this mode appeals managerially as the task duration of maximum likelihood. The Weibull also features more and larger deviations from the mode in the positive direction.

Miller (2003) offers a convenient procedure for specifying the parameters of a three-parameter Weibull distribution from intuitive properties of task duration. We need a value for the duration mode, x_M, (for this, we just use the longest admissible task duration) and a categorization of the risk level as high, medium, or low. Miller suggests high risk for unprecedented tasks, medium for development and some new integration tasks, and low for routine, repetitive, or well-understood tasks. Each risk level is associated with fixed values of two attributes of the task duration that together with with the mode x_M are sufficient to determine all three parameters of the Weibull. Attribute $R_M = x_M/\gamma$ is the ratio of the mode to the minimum duration and $P_M = P(X > x_M)$ is the probability that the duration exceeds the mode. Miller suggests for (risk, R_M, P_M) the values (high, 1.25, 0.8), (medium, 1.20, 0.7), or (low, 1.15, 0.6). PAE concurs.

The three-parameter Weibull distribution can be defined using either triplet (α, β, γ) or (R_M, x_M, P_M). Table 1 shows the mapping between these equivalent descriptions. For example, a medium-risk task with most-likely duration $x_M = 36$ months is endowed with $R_M = 1.20$ and $P_M = 0.7$, and the associated Weibull parameters are:

$$\alpha = \frac{36(1 - 1/1.20)}{[-\ln(0.7)]^{1+\ln(0.7)}} = 11.65,$$

$$\beta = \frac{1}{1 + \ln(0.7)} = 1.554, \text{ and } \gamma = \frac{36}{1.20} = 30.0.$$

In consultation with PAE, we truncate our Weibull at its 90th percentile to avoid unrealistically-long project durations. The maximum allowable duration truncation point is calculated $d_{Max} = \gamma + \alpha[-\ln(0.1)]^{1/\beta}$. Such a Weibull is trivial to generate from a unit-uniform variate U via $X = \gamma + \alpha[-\ln(1 - .9U)]^{1/\beta}$.

We compare the three FCS project plans (baseline, GAO risk first, and GAO C4ISR first) ignoring cost constraints. For each simulated iteration, new task durations are sampled from their Weibull probability distributions and the resulting project completion time is recorded. The simulation is repeated for 60,000 iterations (i.e., we commit about a minute of computing time to each case). We thus induce the random distribution of project completion time for each project plan. Results from these simulations appear in Figure 1.

Table 1. Association between attributes and parameters of the three-parameter Weibull distribution show how to map from attributes to Weibull parameters or vice versa.

Attributes	Parameters
$R_M = \dfrac{x_M}{\gamma}$	$\alpha = \dfrac{x_M(1 - 1/R_M)}{[-\ln(P_M)]^{1+\ln(P_M)}}$
$x_M = \gamma + \alpha\left(1 - \dfrac{1}{\beta}\right)^{1/\beta}$	$\beta = \dfrac{1}{1 + \ln(P_M)}$
$P_M = e^{-(1-1/\beta)}$	$\gamma = \dfrac{x_M}{R_M}$

OPTIMIZE A BUDGET-CONSTRAINED DETERMINISTIC SCHEDULE

Our real-world project has a budget and costs that may be influenced by the rate at which we work to finish tasks. We adopt monthly planning fidelity. For each task, we introduce a set of discretionary task durations where each duration has its own month-by-month cost profile for completing the task. Our total project budget depends on the finish year we choose, where each candidate finish year induces a completely independent set of year-by-year budget guidelines. These generalizations suggest an optimization model to identify the least expensive feasible project completion time. We discretize the starting times for tasks and task durations to months, and to use the following integer linear program to suggest a project schedule:

Index Use [$\sim$cardinality]

$y \in Y$	Fiscal year (alias yh, yf) [$\sim$20]
$i \in I$	Task (alias j) [$\sim$200]
$\ell \in I$	Distinguished, last task in project
$(i, j) \in A$	Pairwise partial order: task i must be completed before task j starts
$m \in M$	Planning month [$\sim$240]
$m \in M(y)$	Month in fiscal year y
$s = s_i \in S_i \subseteq M$	Start month for task i
$d = d_i \in D_i$	Task i duration in months
$1 \le \mathrm{p_i} \le \mathrm{d_i}$	Months since start of ongoing task i

Given Data [units]

$\underline{budget}_{y,yf}$, $\overline{budget}_{y,yf}$	Lower and upper cost range during fiscal year y if program finishes in fiscal year yf [cost]
$cost_{idp}$	Cost of ongoing task i with duration d

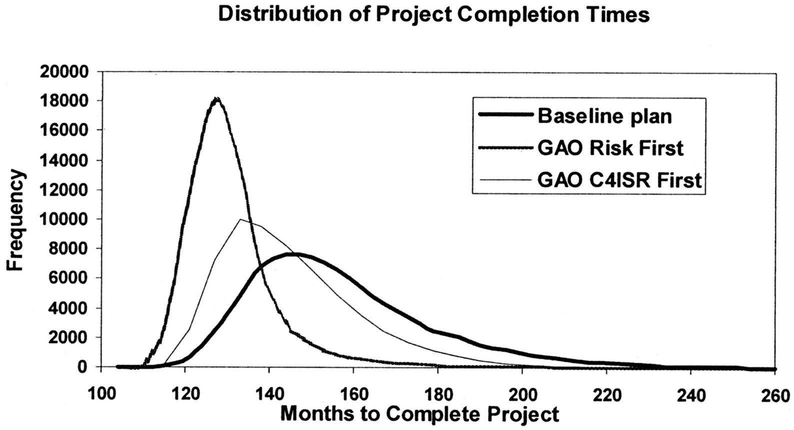


Figure 1. Sixty-thousand samples of each alternate project plan are depicted. There are no cost constraints and each task duration is generated independently from a Weibull distribution reflecting its risk in that plan. "GAO risk first" is the most desirable plan with the highest probability of an early completion time, while the baseline plan has the lowest probability of successful completion at any given time.

	during elapsed month p [cost]
pen_under, *pen_over*	Cost per unit of cumulative budget range violation [months/cost]

Decision Variables [units]

X_{isd}	$= 1$ if task i is started in month s with duration d, 0 otherwise [binary].
Q_{yf}	$= 1$ if finish year of program is year yf, 0 otherwise [binary].
$UNDER_y$,$SLACK_y$,$OVER_y$	When we compare expenditures through fiscal year y with desired lower and upper ranges on total budgets, these variables respectively measure lower-range violation, unspent funds below upper-range, or upper-range violation [months/cost].

Formulation

$$\underset{\substack{X,Q\\UNDER,SLACK,OVER}}{MIN} \sum_{s\in S_\ell,d\in D_\ell\wedge s+d-1\leq\|M\|} (s+d-1)\,X_{\ell sd} + \sum_{y\in Y}(pen\_underUNDER_y + pen\_overOVER_y) \tag{F1}$$

$$s.t. \sum_{s\in S_i,d\in D_i\wedge s+d-1\leq\|M\|} X_{isd} \geq 1 \quad \forall\, i \in I \tag{F2}$$

$$X_{\ell sd} \leq Q_{yf} \quad \forall\, yf \in Y, s \in S_\ell, d \in D_\ell \wedge s + d - 1 \in M(yf) \tag{F3}$$

$$\sum_{yf\in Y} Q_{yf} \leq 1 \tag{F4}$$

$$\sum_{\substack{yh\leq y, m\in M(yh), i\in I, s\in S_i, d\in D_i \\ \wedge m-s+1\geq 1 \wedge m-s+1\leq d \wedge s+d-1\leq \|M\|}} (cost_{id(m-s+1)} X_{isd}) + UNDER_y + SLACK_y - OVER_y = \sum_{\substack{yh\leq y, \\ yf\in Y \wedge yf\geq y}} (\overline{budget}_{yh,yf}) Q_{yf} \quad \forall\, y \in Y \quad \text{(F5)}$$

$$SLACK_y \leq \sum_{\substack{yh\leq y, \\ yf\in Y \wedge yf\geq y}} (\overline{budget}_{yh,yf} - \underline{budget}_{yh,yf}) Q_{yf} \quad \forall\, y \in Y \quad \text{(F6)}$$

$$\sum_{s\in S_i, d\in D_i \wedge s+d-1<s_j} X_{isd} \geq X_{js_jd_j} \quad \forall\, (i,j) \in A,\ s_j \in S_j,\ d_j \in D_j \wedge s_j + d_j - 1 \leq \|M\| \wedge s_j > \underset{s\in S_i d\in D_i}{MIN} (s+d-1) \quad \text{(F7)}$$

$$X_{isd} \in \{0, 1\} \quad \forall\, i \in I,\ s \in S_i,\ d \in D_i \quad \text{(F8)}$$

$$Q_{yf} \in \{0, 1\} \quad \forall\, yf \in Y \quad \text{(F9)}$$

$$UNDER_y \geq 0, SLACK_y \geq 0,\ OVER_y \geq 0 \quad \forall\, y \in Y \quad \text{(F10)}$$

Verbal Description

The overarching goal is to decide how long the project should take to complete. The objective function ($F1$) expresses total planned project duration in months, plus an elastic penalty term for any violation of cumulative budget ranges over the planning horizon. Each partition constraint ($F2$) requires that exactly one start month and duration be selected for each task. Each constraint ($F3$) permits the last project task to be completed in a fiscal year only if that fiscal year has been selected for project completion. Constraint ($F4$) requires that exactly one project completion year be selected. Each constraint ($F5$) accumulates expenditures from the first fiscal year through a current fiscal year and determines whether the cumulative budget ranges have been satisfied, or violated. (This cumulant form is amenable to both a linear programming solver and to managerial interpretation: Brown et al., 1997.) Each constraint ($F6$) limits cumulative slack budget by the hard constraints on yearly program budget determined by finish year. Each constraint ($F7$) ensures, for a pair of tasks adjacent in precedence, that the predecessor task must be completed before the successor task can start. Variable domains are defined by ($F8$–$F10$). ($F8$) can restrict admissible start months for each task and the admissible durations of each task.

TASK DURATIONS AND COSTS

For a task started in month s for duration d months, we can assert any month-by-month cost distribution we want, even including costs for months preceding task start or following task completion (as military research and development often requires: Brown et al., 2004). Here (following explicit guidance from PAE), we simplify: no matter when a task might start for a d-month duration, we allocate its $Task\_Cost_d$ over each month of this duration with a Rayleigh distribution truncated at its 97-th percentile, so that its cost in month p would be:

$$Month\_Cost_p = [Task\_Cost_d/0.97][\exp(\{(p-1)^2 \ln(0.03)\}/d^2) - \exp(\{p^2 \ln(0.03)\}/d^2)].$$

EACH POSSIBLE COMPLETION YEAR HAS ITS OWN BUDGET

The key policy question is (always) "how much are we willing to spend and when are we willing to spend it to finish our project (e.g., by the end of any given future fiscal year)?" Are we willing to spend more for a quicker completion? Are there competing projects that restrict our planned spending pattern? For planning purposes, sooner or later we have to at least estimate upper and lower limits on the overall planned project budget for each financial year of each planned project duration. Here, for any candidate project completion year and budget, we also use a Rayleigh distribution to distribute this budget year-by-year.

A complex, long-term military project rarely meets all its planned budget targets. Sometimes allocated funds are available before they can be used and sometimes costs exceed projections. Accordingly, we accumulate any year-by-year over-expenditure or under-expenditure, but penalize any such cumulative violation year-by-year until the surplus or deficit is repaired. The idea (Brown et al., 2004) is to allow some reasonable flexibility in program management, while showing good faith adhering to overall project budget guidance.

FCS ANNUAL BUDGETS

An FCS project budget estimate has been developed with help from PAE. Separate estimates must be prepared for each feasible project duration, ranging from FY2010 to FY2016. Table 2 shows the minimum, planned and maximum annual budgets for a FY2011 completion that has been Rayleigh-allocated over the planning years.

Preparing budgets for each completion year, we try to follow the best guidance available. For example, a GAO review of FCS (Francis, 2004) concludes that a one-year delay in FCS would increase costs by $4 billion to $5 billion (during the system development and demonstration, and production phases). Relative to the total projected cost of FCS, this represents a 0.5% cost overrun per year of delay. Conversely, Lee (1997) estimates for projects in general that accelerating the pace of work and decreasing a project duration by one year would require an increased budget of 0.2%. Of course, delays in any accelerated plan subject it to cost overruns as well.

SUPERIMPOSE MONTE CARLO SIMULATION OF ANNUAL TASK REVIEWS (WITH POSSIBLE TASK DELAYS AND COST CHANGES) ON SCHEDULE OPTIMIZATION

We nest our cost-constrained project schedule optimization within a simulated annual "project review" of each then-active task. Each reviewed active task may be delayed depending on a probability distribution that depends on the risk of that task, *or on any prior experience with any other task.* The cost of each reviewed task may also change, as can the forecast cost or duration of any future task. Each annual project review is followed by a re-optimization of the remaining future planning horizon. Year-by-year, we conduct an annual project review, re-optimize, and so forth. Figure 2 illustrates how this simulation might progress.

In our simple example, each active task reviewed is delayed with (risk, probability, and delay) of (high, 0.5, 140%); (medium, 0.3, 120%); or (low, 0.2, 110%), and a delayed task's costs

Table 2. For a project completion in FY2011, a nominal total FCS system development and demonstration budget of 20.04 billion 2004 dollars has been Rayleigh-allocated by fiscal year. These planned annual budgets are goals, but the minimum (20%) and maximum (105%) budget ranges are hard constraints. The sums of annual expenditures from FY2003 through any given year are constrained by these cumulative hard constraints. Within these hard cumulative limits, any cumulative expenditure under- or over-plan is penalized and carried forward to the next year, where it will be penalized again if not mitigated.

Year	Minimum Budget ($ Million)	Planned Budget ($ Million)	Maximum Budget ($ Million)
FY2003	$ 175	$ 875	$ 919
FY2004	482	2,410	2,530
FY2005	676	3,382	3,551
FY2006	732	3,658	3,841
FY2007	667	3,335	3,502
FY2008	530	2,652	2,785
FY2009	374	1,871	1,965
FY2010	237	1,183	1,242
FY2011	135	674	708
TOTAL	$4,008	$20,040	$21,042

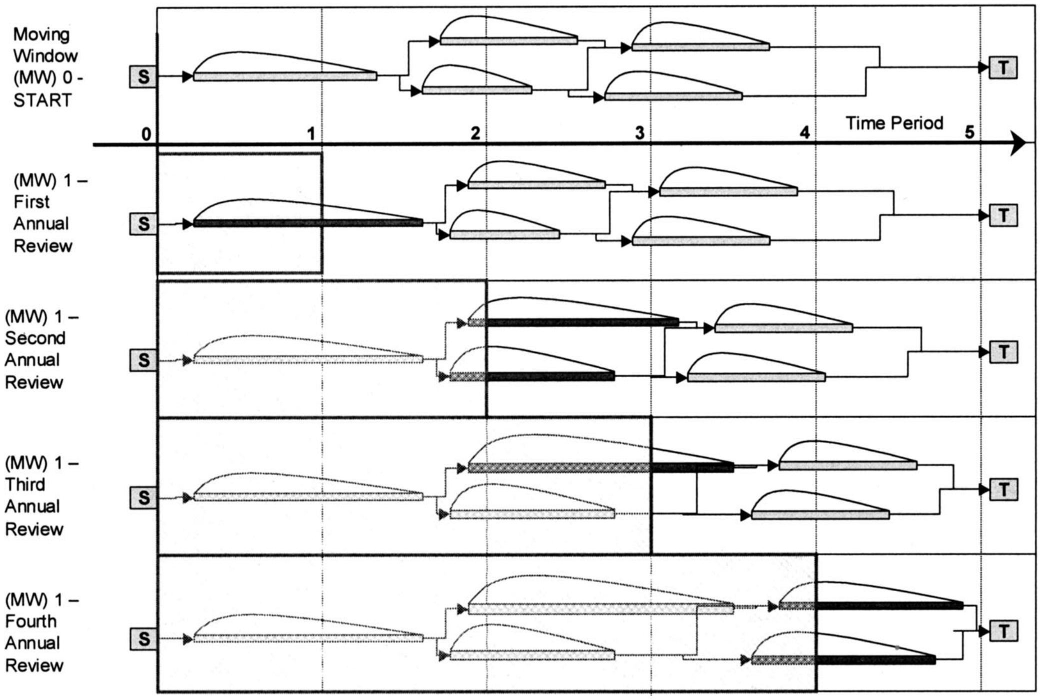


Figure 2. Each annual review (depicted top-to-bottom separating the shaded and un-shaded portions of each timeline row) may delay any currently-active task (i.e., any highlighted task spanning shaded and un-shaded timelines), or change its cost. After each annual review, the remaining schedule is re-optimized with monthly fidelity, subject to annual budget goals induced by the best project duration still achievable. The optimization must complete currently-active tasks as specified by the latest annual review, but can choose any admissible start month for any future task and choose any future admissible task duration it pleases, as long as the associated costs of the chosen duration are bearable. Directed arcs between partially-ordered task pairs and nominal Rayleigh-distributed task budgets are shown to illustrate how the optimization must schedule tasks such that total expenditures follow annual budget guidance.

increase with (risk, probability, change) of (high, 0.5, 150%), (medium 0.3, 130%), or (low, 0.2, 110%). As a practical matter, we permit a task to be delayed to, at most, twice its original duration (longer than this and the task would likely be cancelled, and the project redesigned).

If an annual review extends the remaining optimized project duration, the total project budget changes accordingly (here, it is increased in proportion to the length of the extended duration, though any adjustment is admissible).

This amalgam of annual budget review simulation and optimization of the remaining planning horizon offers a face-valid emulation of actual practice, and our Monte Carlo annual simulation can easily be replaced with a human umpire if more expert control and judgment appeal. We have tested dependent models for inflating costs and task durations, and two key lessons emerge: even mildly inter-task dependent delays cause havoc, and any project overseer would intervene long before these results played out. Although we could model *decreases* in task duration and/or cost, this prospect has never come up with PAE, nor have we ever observed such a signal event in our careers.

IMPLEMENTING THE OPTIMIZATION MODELS

The alternate project plans have been set up in Microsoft Project (2004). We want to use the graphical user interface offered by Project, as well as its integration with the MS Office Suite. Our optimization (with optional Monte Carlo annual reviews) is been implemented in the algebraic modeling language GAMS (Brook et al., 1998).

Each scenario is presented to GAMS as two input scripts, one for tasks, and the other for budgets. The former script has a descriptor for each task specifying each candidate start month, duration in months, and cost. Nobody will use an optimization model they can't control, so this script (via the Project interface) lets a planner completely control alternatives, including "start this task in this month for this duration at this cost."

As you would expect, the GAMS script imports a scenario from Project, solves it, and returns the solution for display and analysis. But, the majority of our GAMS script is devoted to diagnosis and exigent report writing, to better monitor the behavior of our experimental models.

For instance, early experience with our model revealed that although we offer precise controls for task start times and durations, nobody used these: by default, each task can start any time for any admissible duration. As a consequence, an enormous number of alternate task start variables was generated. Solvers mechanically detect and remove redundant model features. However, such "presolve" features do not tell you what they have removed, *or why.* And, presolve will not identify all redundancies: each reduction involves no more than removing one redundant variable with an equation substitution. You can't be sure you have removed the redundancies you worry about unless you filter them out yourself.

So, in addition to the index domain filtering that clutters the summations in our formulation (but makes our intent clear), we formulated an auxiliary, trivial optimization (not displayed) to find the admissible start times and durations for each task.

We work on our formulation and model generator until presolve finds as little as possible left to remove. After such filtering, a typical scenario consists of about 53 thousand constraints, and 19 thousand variables, almost all binary. We would expect such an integer linear program to solve on a laptop in minutes.

We used CPLEX 9.0 (ILOG, 2004). Default CPLEX stalled, and could not find an initial feasible integer solution. We provided an admissible integer starting point from our trivial presolve. CPLEX bogged down in problem preprocessing and integer cut generation. Eventually, to get CPLEX to work, we had do disable most of its default options for cut generation and root node heuristics.

Our solve times are still longer than we expected. If we fix project duration and budget, the resulting optimization model is easier to solve (and we can automate this fixing in GAMS for each project duration we fancy). However, even this simplifying restriction leaves us with a daunting scheduling problem: to choose a start time and duration for each task that satisfies every partial order between tasks, maximally complies with the cumulative budget guidance, and also finishes on time. Typically, it takes us 3 GHz-hours to resolve to a 10% integrality gap.

These integer linear programs may be hard to solve, but they convey remarkable insight we have not gained by any other means. ILP models depend on well-defined assumptions and offer fidelity that closely mimics real-world planning, and they also convey an objective assessment of solution quality that, for instance, lets us confidently compare alternate scenarios.

For instance, the objective assessment of solution quality we get from the integer linear programs is invaluable when comparing two competing alternatives: given assumptions stated clearly, and data defined commensurately, no matter how complex the project, if the optimized solutions exhibit integrality gaps that do not intersect, we can confidently declare a winner.

RESULTS AND CONCLUSION

A Rayleigh-distributed project budget just does not fit the needs of the constituent FCS tasks as the project proceeds for any alternate project plan. Accordingly, we state the budget as a cumulative goal from project start in FY2003 through each year, with any cumulative under- or over-expenditure carried forward to later years, charging a penalty for any deviation from cumulative budget until that violation is rectified. Without this flexibility, we must extend the project finish year and leave Rayleigh-allocated funds for intermediate years unused. For long-term planning, this makes no sense at all. Figure 3 shows how the deterministic, optimized plans use the Rayleigh-distributed project budget, and displays the same expenditures in cumulative terms.

Starting the "baseline plan" in January 2003, we find a (cumulative) cost-constrained schedule that finishes just as quickly as primitive CPM with no cost constraints at all: October 2012. Our nominal task costs and total budget restrictions *just suffice* without delaying the project: this is another reassuring discovery. And, our suggested cumulative expenditure history follows long-term guidance closely.

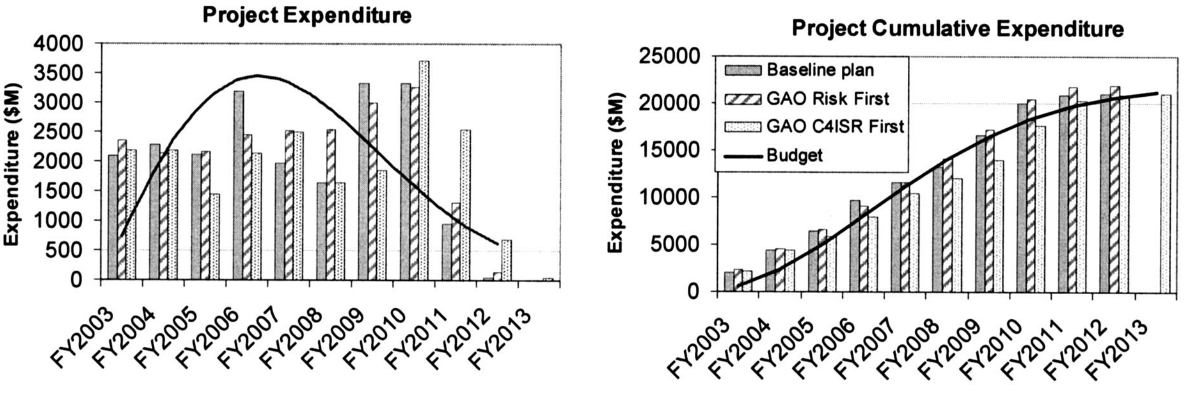
When we simulate annual reviews, with task delays, the optimized project plans take a lot longer to complete (see Figure 4). Monte Carlo delays of task durations as the project proceeds extend achievable project completion, so the projected budget (discovered year-by-year as the project progresses and these delays arise) is characterized by transitions to successively longer finish-year budgets (see Figure 5).

For the baseline plan, just introducing random Monte Carlo task durations increases the median project duration by about 10%. If budget constraints are imposed in addition to random task delays, estimated project duration rises by about 39%. For FCS, a 39% delay corresponds to approximately four years, where a one-year delay has been estimated by the GAO to add between \$4 billion and \$5 billion to the total acquisition cost.

In the absence of budget constraints, mitigating the technologies below the required maturity level prior to other tasks (GAO risk first) leads to project completion faster than the baseline plan. When budget constraints are added, this plan maintains its advantage although it is subject to delays similar to the baseline plan.

Table 3 assembles FCS project duration estimates for each alternate plan and from each of our models. With no budget constraint, it's best to mitigate high-risk technology first. With project budget constraints, both the baseline and risk-first plans are attractive, but with annual review simulation, the GAO C4ISR first plan turns out to be least vulnerable to delay. Given the high risk of the FCS program, we prefer the behavior of GAO C4ISR first.

FCS is a long, complex, technically risky, expensive, and *important* project. But, FCS is not unique in these respects: there are (always) other defense projects that are comparable (Brown et al., 2004). Based on our planning experience with such projects, we recommend a high-level assessment such as that presented here to forecast as early as possible and as well as possible where the fragilities


Project Expenditure

Project Cumulative Expenditure

Figure 3. A Rayleigh-distributed budget for a FY2012 project finish is shown in annual and cumulative terms along with deterministic, optimized expenditures for each alternate project plan. The budget is stated as a cumulative goal from project start in FY2003 through each year, with any cumulative under- or over-expenditure carried forward to later years, charging a penalty for any deviation from cumulative budget goal until that violation is rectified. Note the banking of unused budget (e.g., in FY2007) in anticipation of borrowing it back (e.g., in FY2010). Without this flexibility, we must extend the project years beyond FY2012 and leave allocated funds for intermediate years unused. For long-term planning, this makes no sense at all.

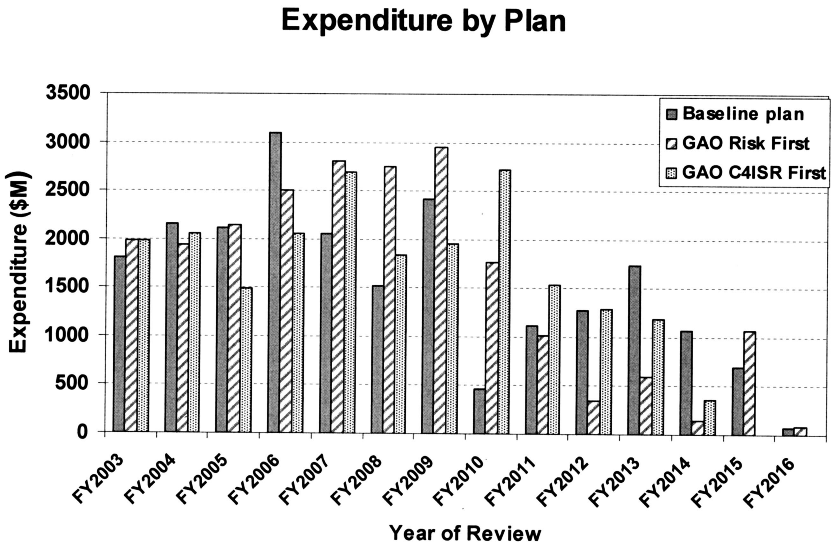


Figure 4. Annual expenditures are shown for optimized FCS project schedules with a simulated annual review of each then-active task that, depending on task risk, may randomly induce a delay and a cost increase. As expected, project completion is delayed for all project plans, and costs rise (by about four years and $600 million, respectively). The idea is to animate how these task delays arise over time and how they cascade and influence other competing or succeeding tasks. Note that GAO C4ISR finishes two years before the other plans.

and vulnerabilities are in an overall project plan, and to prescribe work-arounds sooner, rather than later.

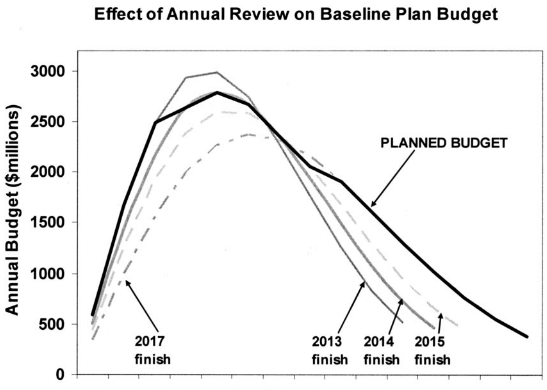


Figure 5. For the baseline plan, as the project progresses year-by-year and is subjected to annual reviews that delay then-active tasks, the remaining project tasks are reoptimized and the project takes longer to complete. This shifts the best achievable project budget to a later year. The display shows when the optimization must jump to a larger and longer budget.

We emphasize key, distinguishing, real-world advantages offered here. *These component models are easy to discuss, illustrate, brief, and understand:* Project scheduling and primitive Monte Carlo simulation are ubiquitous. We co-opt the graphical user interface of a project management system and its data base, and embed the optimizer and Monte Carlo reviews, thereby producing a visually-appealing planning product at a low per-seat cost. The embedding of deterministic optimization within time-phased simulation decouples the two in a way that requires few simplifying assumptions and that invites very basic, intuitive analysis to evaluate results. We closely mimic real-world behavior:

- each optimization decision offers to start a task for some duration on some cost schedule, and this corresponds directly to contract terms we must commit; and
- simulated annual review of each then-active task state can depend on any prior learning, but, more importantly, this dependence can be described in simple, intuitive terms of the facts already in hand for the review.

Table 3. Deterministic CPM gives a lower bound for each plan duration. Monte Carlo CPM, here using for each task an independent Weibull task time based only on that task's risk, shows the delaying influence of task time variability on the median project duration for 60,000 samples of each schedule plan. Deterministic optimized plans honor project budget goals and show the delaying influence of doing so. Optimized plans with Monte-Carlo annual reviews show the combined delaying effects of task time variability and budget goals. For reference, a start in January 2003 for 118 months yields a finish in October 2012.

Schedule Plan	Estimated FCS Program Durations in Months			
	No Budget Constraint		Project Budget Constraint by Fiscal Year Completed, and Allocated Yearly	
	Deterministic CPM	Monte Carlo CPM	Deterministic Optimized	Optimized with Monte Carlo Annual Reviews
Baseline	118	150	118	164
GAO Risk First	116	126	116	162
GAO C4ISR First	129	139	130	145

EPILOG

Time will tell whether our work proves prescient for FCS. We have delivered presentations to the Cost Analysis Improvement Group, Program Analysis and Evaluation, Office of the Secretary of Defense, and thank them, especially Mr. Walter Cooper, for their continued encouragement and support. Grose (2004) exhibits additional underlying detail. Since this writing, a number of revisions to the FCS program and its nominal schedule have already arisen, and these are reported in the open press, where we direct interested readers.

APPENDIX: THREE ALTERNATE PLANS FOR FCS SYSTEMS DEVELOPMENT AND DEMONSTRATION

Summary description tasks are in **bold text** (Microsoft, 2004). Zero-duration tasks are milestones.

Baseline Plan:

ID	Task Name	Estimated Cost ($M)	Duration (Weeks)	Successors
1	Notional Start	0.00	0	24,13,3
2	**Major Events**			
3	Milestone B Complete	0.00	0	4,67,37,29,25,14
4	SFR (System Functional Review)	0.00	0	5,16,26
5	SoS PDR Complete	0.00	0	6,17
6	SoS CDR Complete	0.00	0	7
7	Facilitation	0.00	0	8,95
8	LL IPR Waiver	0.00	0	9,97
9	IPD (Milestone C)	0.00	0	10,77
10	IOC	0.00	0	11,32
11	UA	0.00	0	101
12	**SoS Definition and Design**			
13	Systems Engineering	571.42	104	5
14	Systems Design	1,428.57	260	10
15	**Prototype Systems Build and Test**			
16	1st Variant PDC (Preliminary Design Complete)	0.00	0	17
17	Last Variant PDC (Preliminary Design Complete)	0.00	0	18,20,44
18	Long Lead Prototype	800.00	52	19,21
19	Prototype Integration and Assembly	1,200.00	78	22
20	First Variant CDC (Critical Design Complete)	0.00	0	69,21
21	Last Variant CDC (Critical Design Complete)	0.00	0	22,6
22	Final Prototype	0.00	0	97,8
23	**C4ISR Software and Platform**			
24	SW Build 1	507.93	104	27,44
25	SW Build 2	634.92	130	27,34,69,31,46,52,59
26	SW Build 3	825.39	169	28,52,59
27	SW Build 4	571.42	117	9,63,59
28	SW Build 5	507.93	104	83,89,64
29	SIL Delivery 1 (System Integration Lab)	253.96	52	68,33,30
30	SIL Delivery 2	253.96	52	69,31,27,52
31	SIL Delivery 3	253.96	52	32,28
32	SW Update	190.47	39	11,80
33	Software PDR Complete	0.00	0	34,5
34	Software CDR Complete	0.00	0	6
35	**Integrated Test Program**			
36	**IPS1 (Integration Phase SDD 1)**			
37	SoSIL Development	280.99	51	38,39,30
38	Integration	71.62	13	41,5,40
39	Sims Delivered	0.00	0	40
40	IT and UT	71.62	13	42
41	TRR	0.00	0	42
42	Analysis	71.62	13	45,44
43	**IPS2**			
44	Integration	280.99	51	47,6,46
45	Early Emulators Delivered	0.00	0	46
46	IT/UT	71.62	13	48
47	TRR	0.00	0	48
48	Analysis	71.62	13	50,51,28
49	**IPS3**			
50	Integration	209.36	38	53,52

APPENDIX: Continued

ID	Task Name	Baseline Plan: Estimated Cost ($M)	Baseline Plan: Duration (Weeks)	Successors
51	Initial DP Prime Items Delivered	0.00	0	52
52	IT and UT	71.62	13	54,55
53	TRR	0.00	0	54,55
54	Analysis	104.68	19	58,8
55	User Trial	11.01	2	57
56	**IPS4**			
57	Integration	187.32	34	60,59
58	Initial System Deliveries	0.00	0	59
59	IT and UT	71.62	13	61,63,72
60	TRR	0.00	0	61
61	Analysis	71.62	13	9
62	**IPS5**			
63	Integration	209.37	38	64
64	IMT	71.63	13	65
65	Analysis	71.63	13	77,100
66	**SoS Testing and Integration**			
67	Phase 1: Integration and Test SDD (Simulation)	183.75	78	70,5
68	Phase 2: HW and SW	214.37	91	6,95
69	Phase 3: Prototype	214.37	91	72,57,8
70	Integration and Qualification and Live Fire Tests	489.99	208	73,9,76
71	**Test Events and Milestones**			
72	LUT 1	4.71	2	73
73	LUT 2	4.71	2	77,79,74,98,99
74	IOT (Initial Operational Test) Phase 1	47.11	20	10,75
75	IOT Phase 2	44.76	19	80
76	Integration and Test Production	214.37	91	10,80
77	FUSL	244.99	104	80,11
78	Training and Fielding	244.99	104	80
79	IOTE 1	61.25	26	80
80	IOTE 2	30.62	13	11
81	**Combat Systems Testing**			
82	**Phase 1: LRIP Prime Items**			
83	Integration	634.15	39	85,89,100
84	LRIP PI for SoSIL	0.00	0	85
85	LRIP PI for TFT Delivered	0.00	0	86
86	Testing	211.38	13	87,90
87	Analysis	211.38	13	92,74,79
88	**Phase 2: LRIP Late LRIP PI**			
89	Integration	520.33	32	91
90	LRIP PI for SoSIL	0.00	0	91
91	LRIP PI for TFT Delivered	0.00	0	92
92	Testing	211.38	13	93
93	Analysis	211.38	13	11,10,32
94	**Production**			
95	Facilitation (Pre-LL Production)	682.93	52	100,84,96
96	Facilitation (LL Production)	1,195.12	91	100,84
97	Long Lead Lot 1	682.93	52	98,99,100,9,83,84,76
98	Lot 1	1,024.39	78	79,78
99	Lot 2	1,707.32	130	11,80
100	Lot 3	1,707.32	130	11,80
101	Notional End Task	0	0	

APPENDIX: GAO “Risk First”: Mitigate High Risk Technologies First

ID	Task Name	Estimated Cost ($M)	Duration (Weeks)	Successors
1	Notional Start	0.00	0	57,13,3
2	**Major Events**			
3	Milestone B Complete	0.00	0	4,100,70,62,58,14
4	SFR (System Functional Review)	0.00	0	5,49,59
5	SoS PDR Complete	0.00	0	6,50
6	SoS CDR Complete	0.00	0	7
7	Facilitation	0.00	0	8,128
8	LL IPR Waiver	0.00	0	9,130
9	IPD (Milestone C)	0.00	0	10,110
10	IOC	0.00	0	11,65
11	UA	0.00	0	134
12	**SoS Definition and Design**			
13	Systems Engineering	571.43	104	5
14	Systems Design	1,428.57	260	10
15	**Prototype Systems Build and Test**			
16	**TRL Mitigation (Technology Readiness Level)**			
17	**KPP 1: Joint Interoperability**			
18	Interface and Information Exchange	113.24	65	4
19	**KPP 2: Networked Battle Command**			
20	Security Systems and Algorithms	249.13	143	6
21	Quality of Service Algorithms	67.94	39	3
22	Wideband Waveforms	181.18	104	5
23	Multispectral Sensors and Seekers	90.59	52	3
24	Combat Identification	22.65	13	3
25	Sensor and Data Fusion and Data Compression	67.94	39	3
26	**KPP 3: Networked Lethality**			
27	Dynamic Sensor-Shooter Pairing and Fire Control	90.59	52	3
28	LOS and BLOS and NLOS Precision Munitions Guidance	271.78	156	6
29	Aided Target Recognition	67.94	39	3
30	Auto Target Recognition	181.18	104	5
31	Recoil Management and Lightweight Components	90.59	52	3
32	Distributed Collaboration of Manned and Unmanned Vehicles	226.48	130	5
33	Rapid Battle Damage Assessment	67.94	39	3
34	**KPP 4: Transportability**			
35	High Power Density and Fuel Efficient Propulsion	90.59	52	3
36	**KPP 5: Sustainability and Reliability**			
37	Embedded Predictive Logistic Sensors and Algorithms	90.59	52	3
38	Water Generation and Purification	90.59	52	3
39	**KPP 6: Training**			
40	Computer Generated Forces	22.65	13	3
41	Tactical Engagement Simulation	45.30	26	3
42	**KPP 7: Survivability**			
43	Active Protection System	22.65	13	3
44	Signature Management	90.59	52	3
45	Lightweight hull and Vehicle Armour	10.45	6	3
46	Power Distribution and Control	10.45	6	3
47	Advanced Countermine Technology	226.48	130	5
48	High Density Packaged Power	10.45	6	3
49	1st Variant PDC (Preliminary Design Complete)	0.00	0	50
50	Last Variant PDC (Preliminary Design Complete)	0.00	0	51,53,77

APPENDIX: Continued

ID	Task Name	Estimated Cost ($M)	Duration (Weeks)	Successors
107	IOT (Initial Operational Test) Phase 1	47.11	20	10,108
108	IOT Phase 2	44.76	19	113
109	Integration and Test Production	214.37	91	10,113
110	FUSL	244.99	104	113,11
111	Training and Fielding	244.99	104	113
112	IOTE 1	61.25	26	113
113	IOTE 2	30.62	13	11
114	**Combat Systems Testing**			
115	**Phase 1: LRIP Prime Items**			
116	Integration	634.15	39	118,122
117	LRIP PI for SoSIL	0.00	0	118
118	LRIP PI for TFT Delivered	0.00	0	119
119	Testing	211.38	13	120,123
120	Analysis	211.38	13	125,107,112
121	**Phase 2: LRIP Late LRIP PI**			
122	Integration	520.33	32	124
123	LRIP PI for SoSIL	0.00	0	124
124	LRIP PI for TFT Delivered	0.00	0	125
125	Testing	211.38	13	126
126	Analysis	211.38	13	11,10
127	**Production**			
128	Facilitation (Pre-LL Production)	833.33	65	129
129	Facilitation (LL Production)	1,166.67	91	133,117
130	Long Lead Lot 1	666.67	52	131,132,133,9,116,117
131	Lot 1	1,000.00	78	112,111
132	Lot 2	1,666.67	130	11,113
133	Lot 3	1,666.67	130	11,113
134	Notional End Task	0.00	0	

APPENDIX: GAO "C4ISR First": Develop C4ISR Infrastructure First

ID	Task Name	Estimated Cost ($M)	Duration (Weeks)	Successors
1	Notional Start	0	0	24,13,3
2	**Major Events**			
3	Milestone B Complete	0	0	4,67,37,29,25,14
4	SFR (System Functional Review)	0	0	5,26
5	SoS PDR Complete	0	0	6,16
6	SoS CDR Complete	0	0	7,17
7	Facilitation	0	0	8,95
8	LL IPR Waiver	0	0	9,97,21
9	IPD (Milestone C)	0	0	10,77
10	IOC	0	0	11,32
11	UA	0	0	101
12	**SoS Definition and Design**			
13	Systems Engineering	571.43	104	5
14	Systems Design	1428.57	260	10
15	**Prototype Systems Build and Test**			
16	1st Variant PDC (Preliminary Design Complete)	0	0	17
17	Last Variant PDC (Preliminary Design Complete)	0	0	18
18	Long Lead Prototype	800	52	19,20,21
19	Prototype Integration and Assembly	1200	78	22
20	First Variant CDC (Critical Design Complete)	0	0	95
21	Last Variant CDC (Critical Design Complete)	0	0	22
22	Final Prototype	0	0	57,69,97,96
23	**C4ISR Software and Platform**			
24	SW Build 1	507.94	104	27,44
25	SW Build 2	634.92	130	27,34,69,31,46,52,59
26	SW Build 3	825.4	169	28,52,59
27	SW Build 4	571.43	117	9,63,59
28	SW Build 5	507.94	104	83,89,64
29	SIL Delivery 1 (System Integration Lab)	253.97	52	68,33,30
30	SIL Delivery 2	253.97	52	69,31,27,52
31	SIL Delivery 3	253.97	52	32,28
32	SW Update	190.48	39	11,80
33	Software PDR Complete	0	0	34,5
34	Software CDR Complete	0	0	6
35	**Integrated Test Program**			
36	**IPS1 (Integration Phase SDD 1)**			
37	SoSIL Development	280.99	51	38,39,30
38	Integration	71.63	13	41,5,40
39	Sims Delivered	0	0	40
40	IT and UT	71.63	13	42
41	TRR	0	0	42
42	Analysis	71.63	13	45,44
43	**IPS2**			
44	Integration	280.99	51	47,6,46
45	Early Emulators Delivered	0	0	46
46	IT and UT	71.63	13	48
47	TRR	0	0	48
48	Analysis	71.63	13	50,51,28
49	**IPS3**			
50	Integration	209.37	38	53,52
51	Initial DP Prime Items Delivered	0	0	52
52	IT and UT	71.63	13	54,55
53	TRR	0	0	54,55
54	Analysis	104.68	19	58,8

APPENDIX: Continued

ID	Task Name	Estimated Cost ($M)	Duration (Weeks)	Successors
55	User Trial	11.02	2	57
56	**IPS4**			
57	Integration	187.33	34	60,59
58	Initial System Deliveries	0	0	59
59	IT and UT	71.63	13	61,63,72
60	TRR	0	0	61
61	Analysis	71.63	13	9
62	**IPS5**			
63	Integration	209.37	38	64
64	IMT	71.63	13	65
65	Analysis	71.63	13	77,100
66	**SoS Testing and Integration**			
67	Phase 1: Integration and Test SDD (Simulation)	183.75	78	70,5
68	Phase 2: HW and SW	214.37	91	6,95,57
69	Phase 3: Prototype	214.37	91	72
70	Integration and Qualification and Live Fire Tests	489.99	208	73,9,76
71	**Test Events and Milestones**			
72	LUT 1	4.71	2	73
73	LUT 2	4.71	2	77,79,74,98,99,76
74	IOT (Initial Operational Test) Phase 1	47.11	20	10,75
75	IOT Phase 2	44.76	19	80
76	Integration and Test Production	214.37	91	10,80
77	FUSL	244.99	104	80,11
78	Training and Fielding	244.99	104	80
79	IOTE 1	61.25	26	80
80	IOTE 2	30.62	13	11
81	**Combat Systems Testing**			
82	**Phase 1: LRIP Prime Items**			
83	Integration	634.15	39	85,89,100
84	LRIP PI for SoSIL	0	0	85
85	LRIP PI for TFT Delivered	0	0	86
86	Testing	211.38	13	87,90
87	Analysis	211.38	13	92,74,79
88	**Phase 2: LRIP Late LRIP PI**			
89	Integration	520.33	32	91
90	LRIP PI for SoSIL	0	0	91
91	LRIP PI for TFT Delivered	0	0	92
92	Testing	211.38	13	93
93	Analysis	211.38	13	11,10,32
94	**Production**			
95	Facilitation (Pre-LL Production)	682.93	52	100,84,96
96	Facilitation (LL Production)	1195.12	91	100,84
97	Long Lead Lot 1	682.93	52	98,99,100,9,83,84,76
98	Lot 1	1024.39	78	79,78
99	Lot 2	1707.32	130	11,80
100	Lot 3	1707.32	130	11,80
101	Notional End Task	0	0	

REFERENCES

Ahuja, R.K., Magnanti, T.L., and Orlin, J.B. (1993). *Network Flows: Theory, Algorithms and Applications,* Prentice Hall, Englewood Cliffs, NJ.

Babbar, M.M., Tintner, G., and Heady, E.O. (1955). "Programming with Consideration of Variations in Input Coefficients," *Journal of Farm Economics,* May, **37,** No. 2, pp. 333–341.

Bowman, E.H. (1958). "The Schedule Sequencing Problem," *Operations Research,* **7,** pp. 621–624.

Brady, E. (2003). *Future Combat Systems,* Modeling and Simulation Conference, Project Executive Office Ground Combat Systems, U.S. Army, 30 January 2003. Accessed 14 January 2004 via the World Wide Web at *http://www.boeing.com/defense-space/ic/fcs/bia/modsim_030130.html.*

Brook, A., Kendrick, D., Meeraus, A., and Raman, R. (1998). *GAMS: A User's Guide,* The Scientific Press, Washington, D.C.

Brown, G.G., Dell, R.F. and Newman, A.M (2004). "Optimizing Military Capital Planning," *INTERFACES,* **34,** pp. 415–425.

Brown, G.G., Dell, R.F., and Wood, R.K. (1997). "Optimization and Persistence," *INTERFACES,* **27,** pp. 15–37.

Demeulemeester, E., and Herroelen, W. (2002). *Project Scheduling: A Research Handbook,* Volume 49 of the International Series in Operations Research and Management Science, Kluwer Academic Publishing, Boston.

Dodin, B. (1984). "Bounding the Project Completion Time Distribution in PERT Networks," *Operations Research,* **33,** pp. 862–881.

Francis, P.L. (2004). "Defense Acquisitions: The Army's Future Combat Systems' Features, Risks and Alternatives," GAO-04-635T, Testimony before the Subcommittee on Tactical Air and Land Forces, Committee on Armed Services, U.S. House of Representatives, April.

Fulkerson, D.R. (1962). "Expected Critical Path Lengths in PERT Networks," *Operations Research,* **10,** pp. 808–817.

Grose, R.T. (2004). "Cost-constrained Project Scheduling with Task Durations and Costs That May Increase Over Time: Demonstrated with the U.S. Army Future Combat Systems," MS Thesis in Operations Research, Naval Postgraduate School, Monterey, CA, March.

Gutjahr, W.J., Stauss, C., and Wagner, E. (2000). "A stochastic branch and bound approach to activity crashing in project management," *INFORMS Journal on Computing,* **12,** No. 2, pp. 125–135.

Herroelen, W., and Leus, R. (2004). "Robust and Reactive Scheduling: A Review and Classification of Procedures," *International Journal of Production Research,* **42,** No. 8 (15 April), pp. 1599–1620.

Herroelen, W., Reyck, B.D., and Demeulemeester, E. (1998). "Resource-Constrained Project Scheduling: A Survey of Recent Developments," *Computers & Operations Research,* **25,** No. 4, pp. 279–302.

Igelmund, G., and Radermacher, F.J. (1983). "Pre-selective strategies for the optimization of stochastic project networks under resource constraints," *Networks,* **13,** pp. 1–28.

ILOG (2004). ILOG Optimization Suite White Paper. Accessed 13 June 2004 via the World Wide Web at *http://www.ilog.com/products/optimization/papers.cfm.*

Kelly, J.E. (1961). "Critical Path Planning and Scheduling: Mathematical Basis," *Operations Research,* **9,** pp. 296–320.

Kelly, J.E. (1963). "The Critical Path Method: Resource Planning and Scheduling," in Muth, J.F., and Thompson, G.L. (Eds.), *Industrial Scheduling,* Prentice Hall, Englewood Cliffs.

Law, A.M. and Kelton, W.D. (2000). *Simulation Modeling and Analysis,* third ed., McGraw-Hill, New York.

Lee, D. (1997). *The Cost Analyst's Companion,* Logistics Management Institute, McLean, VA.

Malcolm, D.G., Roseboom, J.H., Clark, C.E., and Fazar, W. (1959). "Application of a Technique for Research and Development Program Evaluation," *Operations Research,* **7,** pp. 646–669.

Microsoft Corporation (2004). "Microsoft Office." Accessed 10 February 2004 via the World Wide Web at *http://office.microsoft.com/home/office.aspx?assetid=FX01085795.*

Miller, S. (2003). "Simulation Based Schedule Analysis," In *Proceedings of the Department of Defense Cost Analysis Symposium,* Washington, D.C. Accessed 12 December 2003 via the World Wide Web at *http://www.ra.pae.osd.mil/adodcas/index2.htm.*

Pritsker, A.B., Watters, L.J., and Wolfe, P.M. (1969). "Multi-Project Scheduling with Limited Resources, A Zero-One Programming Approach," *Management Science,* **16,** pp. 93–108.

Sengupta, J.K., Tintner, G., and Morrison, B. (1963). "Stochastic Linear Programming with Applications to Economic Models," *Economica,* **30,** pp. 262–275.

Senju, S., and Toyoda, Y. (1968). "An Approach to Linear Programming with 0–1 Variables," *Management Science,* **15,** No. 4, pp. 196–207.

Sun Microsystems (2005). "Java technology." Accessed 28 May 2005 via the World Wide Web at *http://java.sun.com.*

Tintner, G. (1955). "Stochastic Linear Programming with Applications to Agricultural Economics," In Antosiewicz, H.A., *Second Symposium on Linear Programming,* National Bureau of Standards, Washington, D.C., 27–29 January.

Tintner, G. (1960). "A Note on Stochastic Linear Programming," *Econometrica,* **28,** No. 2, pp. 490–495.

U.S. General Accounting Office (2003). *"Issues Facing the Army's Future Combat Systems Program,"* GAO-03-1010R, Washington, D.C., August.

Wiest, J.D. (1964). Some Properties of Schedules for Large Projects with Limited Resources, *Operations Research,* **12,** pp. 395–418.

Welch, L.D. (2003). *Objective Force and Future Combat Systems Independent Assessment Panel Report*, Institute for Defense Analysis, Alexandria, VA.

Wynne, M.W. (2003). *Operation of the Defense Acquisition System,* DoD Instruction 5000.2, Office of the Principal Deputy Under Secretary of Defense for Acquisition, Technology and Logistics, U.S. Department of Defense, 12 May.

Yang, B., Geunes, J., and O'Brien, W.J. (2001). *Resource-Constrained Project Scheduling: Past Work and New Directions,* Research Report 2001–6, Department of Industrial and Systems Engineering, University of Florida.

Military Operations Research Society (MORS) Oral History Project Interview of Gregory S. Parnell, FS

Dr. Bob Sheldon, FS

Group W, Inc
bs@group-w-inc.com

MILITARY OPS RESEARCH HERITAGE ARTICLE

ORAL HISTORY DISCLAIMER

Oral Histories represent the recollections and opinions of the person interviewed, and not the official position of MORS. Omissions and errors in fact are corrected when possible, but every effort is made to present the interviewee's own words.

INTRODUCTION

Greg Parnell was MORS President in 1993–1994, was elected a Fellow of MORS in 1997, and was the Clayton Thomas Laureate in 2002. He was Editor of the *Military Operations Research* Journal from 1996 to 2001. Greg is currently a professor of systems engineering at the United States Military Academy at West Point and a senior principal at Innovative Decisions Inc. This interview was conducted in Greg's office in Mahan Hall, West Point on 21 June 2005.

BOB SHELDON: Today is the 21st of June, 2005 and we're here at Greg Parnell's office at West Point for an oral history interview. First of all, tell us where you were born and raised.

GREG PARNELL: Before we start, I am honored to be interviewed and pleased to learn that you have started a youth movement in the MORS heritage program! I was born in Rochester, New York, and I was raised in several small towns in and around the city.

BOB SHELDON: Where did you go to high school?

GREG PARNELL: I went to two high schools: Avon High School and Caledonia-Mumford High School. Both are south of Rochester.

BOB SHELDON: Give me your parents' names.

GREG PARNELL: George Samuel Parnell and Mary Church Parnell.

BOB SHELDON: Did they influence your decision to study mathematics and operations research?

GREG PARNELL: No.

BOB SHELDON: Did you take an early interest in math?

GREG PARNELL: Yes, I was interested in math, physics, and chemistry. My high school math teacher helped me develop my abilities in algebra, trigonometry and geometry.

BOB SHELDON: Where did you go to college?

GREG PARNELL: For undergraduate studies, I attended State University of New York at Buffalo and majored in aerospace engineering. I did my masters at University of Florida in industrial and systems engineering. I did a part time degree in systems management at University of Southern California. Finally, I did my Ph.D. at Stanford in engineering-economic systems. The department is now called Management Science and Engineering.

BOB SHELDON: Were you an ROTC scholarship student?

GREG PARNELL: Yes. I received one of the first three-year Air Force ROTC scholarships. This was before the Air Force started the four-year scholarship program. After completing the program as a distinguished graduate, I received a regular commission in the Air Force in 1970.

BOB SHELDON: Going from your bachelors to your masters, how did you decide on your field?

GREG PARNELL: That's an interesting story. I was an aerospace engineering undergraduate. I was working in space operations and I wanted to get an MBA. The Air Force needed engineers and wanted to send me for an engineering degree. I wasn't real sure what I was getting into, but industrial and systems engineering sounded useful.

BOB SHELDON: Did you have other assignments before getting your masters?

GREG PARNELL: I had three one-year assignments before my masters program. Two space operations assignments; one in the Space Operations Center in Colorado Springs, one at a Spacetrack Site in Diyarbakir, Turkey. Then I had my first of several acquisition management assignments at Wright Patterson Air Force Base while I was waiting for my school slot.

BOB SHELDON: What was your career field?

GREG PARNELL: I was originally in the space operations career field. When I was in Air Force ROTC, I looked at the assignment book and it said "Space operations officers command and direct sensors throughout the world" so I thought that was pretty good. When I got on active duty, I found out that "throughout the world" included Alaska and Greenland! Early on, I learned the decision analysis principal of the Value of Information.

BOB SHELDON: Did you volunteer to go to Turkey right away, or was that a normal rotation?

GREG PARNELL: I went to Colorado Springs on a four-year controlled tour. However, my commander released his junior officers to go to remote after one year. So I had a choice of going to Shemya, Alaska; Clear, Alaska; Thule, Greenland, or Diyarbakir, Turkey. Turkey was the best trade-off of mission and location. This was one of my first multiple objective decisions!

BOB SHELDON: How did you like Turkey?

GREG PARNELL: It was very interesting. Someone told me that if you want to appreciate Turkey, you have to appreciate different shades of brown. We were in a remote area, about 700 miles from the Russian border. The nearest city was Diyarbakir, the home of Mustafa Kemal Ataturk, the famous leader of Turkey. I was able to travel to some of the surrounding area. One of the interesting trips was to the Mt Ararat region.

BOB SHELDON: Where did you go after Turkey?

GREG PARNELL: I went to my first acquisition management assignment at Wright Patterson Air Force Base, Ohio. I spent 10 years in acquisition management assignments, at Aeronautical Systems Division and the Ballistic Missile Office. Then I got my Ph.D. and did operations research assignments, including teaching twice at the Air Force Institute of Technology (AFIT) and an assignment at Air Force Studies and Analysis in the Pentagon.

BOB SHELDON: What did you do in your first acquisition assignment?

GREG PARNELL: I was a subsystem integrator in the Deputy for Subsystems of the Aeronautical Systems Division. My job was to coordinate government furnished equipment that was manufactured and delivered for installation by defense contractors into the F-111, A-7, F-5, and F-4 aircraft.

BOB SHELDON: What problems did you deal with?

GREG PARNELL: The biggest challenge was providing the equipment to meet the aircraft contractors need dates. The government was at risk if government subsystems were not delivered on time for the prime contractors to meet their aircraft delivery dates. Through lots of phone calls, onsite visits, and briefings to managers, we were able to get the subsystem deliveries back on schedule.

I was very fortunate that I got operational experience. In operations you learn to get the job done as best as you can. Operations gave me a real-time focus. In Turkey, I was in charge of changing all the procedures and tactics for a new system. So I had to deal with the acquisition community. When I got to the acquisition community, I better understood acquisition management. Acquisition gave me experience thinking like a decision-maker. The operational and acquisition experiences were of great value in my subsequent analysis and teaching assignments.

BOB SHELDON: Did you push to get to grad school or did one of your mentors encourage you?

GREG PARNELL: I wanted to eventually go to grad school; but, I was not ready in my senior year. After a couple years in the Air Force, I applied for AFIT.

BOB SHELDON: What kind of curriculum did you take in your first masters?

GREG PARNELL: It was an 18 month industrial and system engineering program. I took courses in systems engineering, operations research, and industrial engineering. My masters project was a range scheduling simulation for an organization at Eglin Air Force Base, Florida.

BOB SHELDON: How did you choose Florida for grad school?

GREG PARNELL: I chose some MBA schools but AFIT chose an engineering program for me. My boss, then Lt Col Hank Passi, talked them out of that school. Then AFIT offered me University of Florida. In addition to a good location, they had a great program and great young faculty.

BOB SHELDON: Any notable professors?

GREG PARNELL: Yes, there were several. Mike Thomas was the department head. He later was Director of the School of Industrial Systems Engineering and then Provost at Georgia Tech. Other well known faculty were Thom Hodges, Don Ratliff, Don Hearn, and Richard Francis. Thom Hodges was my academic and masters project advisor. Thom later became department head and chaired professor at North Carolina State.

BOB SHELDON: Were you part of the acquisition career field at the time?

GREG PARNELL: Yes.

BOB SHELDON: What was your assignment after the masters program?

GREG PARNELL: After my masters program, I went back to Wright Patterson Air Force Base to work for Col Hank Passi, who directed a system program office. I worked on sound suppressors for the F-15 aircraft and then became special projects officer for the director.

BOB SHELDON: Did your grad school in industrial and systems engineering help you in your acquisition positions?

GREG PARNELL: Yes. Not so much in my first job, but the next job at the Ballistic Missile Office and especially when I became chief of systems engineering. As chief of systems engineering, I had responsibility for missile performance and life cycle cost. For example, one of the problems we worked was to calculate the probability that a missile launch accident could result in a death. We developed a model to calculate that probability. We also developed the life cycle cost model used for all system trade studies.

BOB SHELDON: Did you build some of those models yourself or did other people build them for you?

GREG PARNELL: The Aerospace Corporation was my systems engineering contractor. They built and operated the models. In one case, I had to redo the probabilistic analysis because it was challenged.

BOB SHELDON: When did you start working on ballistic missiles?

GREG PARNELL: I started in 1978 and spent four-and-a-half years at the Ballistic Missile Office. I worked on the Mark 12A reentry vehicle for the Minuteman III missile, the MX transporter, and Peacekeeper in Minuteman silos.

BOB SHELDON: Were there any interesting challenges you faced?

GREG PARNELL: Yes. As a major-select, I managed the MX transporter, the vehicle that moved the missile between multiple protective structures. My program budget was $3.3 billion in 1980 dollars. The Director of Engineering told me I had two problems: making sure the transporter worked on desert roads and reducing the life cycle cost.

The mobility challenge was getting a vehicle that weighed over a million pounds to work in the desert on low cost roads. We had a program to test the mobility. The specially developed tires failed very early in the test. We tried commercial off-the-shelf tires (used for large mining trucks) and they worked great. The commercial tires would save money, since the roads did not have to be significantly improved. We also changed the engine to reduce costs and increase reliability.

BOB SHELDON: What was the reason for developing special tires?

GREG PARNELL: We wanted to save money in shelter construction. The tire height drove shelter construction costs. Commercial tires and reduced road construction costs offset the shelter construction costs.

BOB SHELDON: What happened to the MX program?

GREG PARNELL: The MX program was cancelled by President Reagan. The missile was named Peacekeeper and placed in Minuteman silos.

BOB SHELDON: What was your job after the MX transporter?

GREG PARNELL: I became the Chief of Missile Systems Engineering for the Peacekeeper missile. During that job, I applied to get my Ph.D.

BOB SHELDON: How were you sponsored for your Ph.D.?

GREG PARNELL: I was centrally selected late in the cycle. I had two mentors that told me not to go get a Ph.D. They recommended I be a career acquisition officer. I had two other mentors (with Ph.D.s), that recommended I get a Ph.D.

Although I enjoyed project management, I enjoyed the decision analysis more. I liked to analyze major decisions and present my recommendations to senior leaders more than the day-to-day management. So, I decided to apply. The Air Force told me, “Sorry, we don’t have any slots for you.” But in late January, someone canceled and I got the slot. I applied to Stanford, MIT, and Georgia Tech. I was fortunate to go to Stanford.

BOB SHELDON: Normally, the Air Force doesn't like to spend money on big tuition colleges. How did you get to Stanford?

GREG PARNELL: I requested Stanford. I called a captain at AFIT that approved the school assignments and he was concerned about the high cost. He put me on hold, came back five minutes later and said you can go to Stanford!

BOB SHELDON: Where were your assignments on the ballistic missile program?

GREG PARNELL: The Ballistic Missile Office was in California, at the old Norton Air Force Base before it closed.

BOB SHELDON: So you stayed out in California at Norton?

GREG PARNELL: Yes, we were there four-and-a-half years and then we went up to Northern California for three years.

BOB SHELDON: How did you narrow it down to the three colleges you applied to?

GREG PARNELL: I selected the three schools by calling every operations research school AFIT used. After talking to them about their program, I asked them which three schools were the best. Three schools were mentioned many times: Georgia Tech, MIT, and Stanford. I knew Georgia Tech because I knew the professors there. Two of the professors taught me at Florida. Then I went to MIT in February; the wind howling off the Charles River was unbelievable. I was impressed with their program.

The next week, I flew out to Stanford. The sun was out and it was a spectacular day. I talked to Professor Ronald Howard, one of the fathers of decision analysis. In my meeting with him, the first thing he said was, "How can I help you make your decision?" I was also interested in doing research at the Center for International Security and Arms Control (CISAC). It was a combination of great school, great professors, great opportunities, and great weather!

BOB SHELDON: Tell us about some of your notable professors at Stanford.

GREG PARNELL: The person that has been the most influential was Howard. Ron was on my committee. Ron is still teaching and I see him regularly at professional meetings. My advisor was Don Dunn. Don taught public policy, was interested in arms control, and was a member of the CISAC. He taught me that research was asking the right questions. I was fortunate enough to have as my third committee member Alexander George, a well known political scientist. Recently he became emeritus professor. Also, Condi Rice was an assistant director at CISAC. I was a teaching assistant for one of her courses.

BOB SHELDON: Did you bump into George Dantzig?

GREG PARNELL: I attended talks that he gave, but I never took one of his classes. He was well respected at Stanford.

BOB SHELDON: What was your thesis topic at Stanford?

GREG PARNELL: I did my dissertation on nuclear arms control during the height of the nuclear arms build up (Parnell, "Large Bilateral Reductions in Superpower Nuclear Weapons," Ph.D. Dissertation, Stanford University, July 1985). We looked at large bilateral reductions in superpower nuclear weapons. We examined the incentives for the two superpowers to comply with or violate the treaty.

BOB SHELDON: How did you approach your dissertation technically?

GREG PARNELL: The general area was treaty verification. I researched the early writing on verification. We used decision analysis and cooperative game theory. The key issues were technology and information. If both superpowers mutually agreed to do away with nuclear weapons, the nation with the best information and best technology could recover if the other nation violated the agreement.

BOB SHELDON: When you were finishing up at Stanford, what kind of a job were you looking for?

GREG PARNELL: As I started looking for an operations research job, an officer at the Operational Sciences Department contacted me about teaching at AFIT. They were looking for somebody that had some space experience to lead their Space Operations Program and teach operations research. I decided I would like to teach.

BOB SHELDON: What year did you finish at Stanford?

GREG PARNELL: I finished in 1985 and went to AFIT.

BOB SHELDON: What courses did you teach?

GREG PARNELL: I taught operations research (introduction to management science, decision analysis, and simulation) and artificial intelligence courses. I was there three years and then went to the National Defense University (NDU) as a research fellow. I also attended the Industrial College of the Armed Forces, class of 1989.

BOB SHELDON: Did you develop any new courses?

GREG PARNELL: I developed an artificial intelligence and operations research course.

BOB SHELDON: Decision analysis has caught on at AFIT. I see it still continues to be a popular part of their curriculum. How did you pick out the textbook and what you wanted to teach?

GREG PARNELL: Joe Tatman was a Ph.D. student with me at Stanford. Joe came to the Math Department and started the decision analysis course during my first assignment. When I came back as Operational Sciences department head in 1993, I introduced the decision analysis sequence in the Operational Sciences Department.

I used Bob Clemen's *Making Hard Decisions* (2nd Edition, Duxbury Press, 1996) for the introductory decision analysis course. It was a new book at the time. I also taught an advanced decision analysis and a decision analysis practice course (using Professor Howard's approach at Stanford). I first started teaching multiple objective decision analysis at Virginia Commonwealth University in 1996. I used Kirkwood's *Strategic Decision Making: Multiobjective Decision Analysis with Spreadsheets* (Duxbury Press, 1997). I gave my notes to Jack Jackson, who taught decision analysis when I left. Jack Kloeber, Stephen Chambal, and others have taught decision analysis at AFIT. Dick Deckro has also been a major part of the decision analysis program. His students have applied Value-Focused Thinking and multiple objective decision analysis in many important problem domains.

BOB SHELDON: Did you consider being selected for ICAF a good thing?

GREG PARNELL: Yes. I was selected as an NDU fellow. The fellows then attend either ICAF or National War College. My acquisition experience put me in ICAF. I applied because I thought it would be more research oriented and I wanted work on nuclear force analysis. I was excited about going to Washington. Five years later, I was excited about leaving!

BOB SHELDON: How was your year at ICAF?

GREG PARNELL: It was a great year. I thought I was on an athletic scholarship. I played softball, volleyball, and soccer. My ICAF group studied the nuclear industry. The highlight was the trip to the Soviet Union in the spring of 1989. We were the first NDU group to go to the Soviet Union in many years. Twenty of us visited Moscow, Tbilisi (Georgia), Volgograd, and St. Petersburg.

1989 was a dramatic time in Russian history. When the Secretary of Defense was briefing us, one of the students who wanted to go to the Soviet Union raised his hand and said, "Can we go to the Soviet Union this year?" A week later we found out we could. Since my research project was related to strategic nuclear arms control, I was selected to go.

BOB SHELDON: Did you meet some of your Soviet counterparts?

GREG PARNELL: Yes, we did. We met Soviet military and civilians in the four major cities.

BOB SHELDON: What were your impressions?

GREG PARNELL: Russia has a wonderful history and culture. Unfortunately, my impression at the time was that the only thing that worked was the military. The military was very professional. In uniform, we attended the WWII victory celebration in Volgograd. It was a moving ceremony. We were treated like celebrities. The lines at stores were what we expected. The best buildings were built before 1917. The people were in turmoil. One medical student told us, "the only reason you would come to visit us is to laugh at us." We left the Soviet Union on a train from St. Petersburg to Finland. When we crossed the border, everyone on the train cheered and applauded. I wondered what the Russian train employees and guards must have thought.

BOB SHELDON: Did you have a Pentagon assignment lined up while you were at ICAF?

GREG PARNELL: I wanted to stay five years in DC to get my daughter through high school. As a lieutenant colonel, I was hired by Air Force Studies and Analyses Agency (AFSAA) as a deputy division chief of the Force Analyses Division in the Strategic Forces Directorate.

BOB SHELDON: Who did you interview with in Studies and Analysis? Maj Gen Harrison?

GREG PARNELL: No, this was during Maj Gen Alexander's tenure. The colonel who hired me retired before I got there. Since I was the senior lieutenant colonel in the division, I became the acting division chief. At the time, all the AFSAA division chiefs were rated. In my first couple of weeks, we did a successful study for General Larry Welch, the Air Force Chief of Staff. The day after the briefing, my Director, Col Knox Bishop told me "You're now the division chief."

BOB SHELDON: That first study you worked on, did you use some of your OR experience?

GREG PARNELL: Yes, our major model was called the Arsenal Exchange Model, developed by Bill Cotsworth. I had Bert Head (now at NSA) as my deputy division chief and a small group of analysts. The Arsenal Exchange Model used goal programming to optimize nuclear force allocations. (It has subsequently been modified by Bill for conventional forces.) Fortunately, I had taught goal programming at AFIT.

At the time all the strategic force analysts (in different organizations and even in the same organizations) used different strategic force databases! As a result, we could not easily compare the results of studies without a lot of work. We did an interesting study of strategic force models involving the Joint Staff, OSD, Strategic Air Command, AFSAA, and RAND strategic nuclear models. All the models had different algorithms. We wanted to compare the results on standard problems. As we standardized the data and the analysis assumptions, the model results converged to the same answers! I learned a valuable lesson. The three most important analysis variables are the analyst, the data, and the model—in that order.

BOB SHELDON: Were all of those optimization models?

GREG PARNELL: Yes, they were all optimization models but with different techniques.

BOB SHELDON: What other projects did you work on?

GREG PARNELL: One of my bosses, Maj Gen George Harrison, had a wonderful saying. He said, "You can educate in advance, you can support the decision maker in real time, or you can analyze the fallout." I tried to get my division prepared to educate in advance or support in real time. We did a lot of work to support the Air Staff, which supported DoD, the Joint Staff, and U.S. Strategic Arms Reduction Treaty (START) negotiators.

One of the most interesting *support the decision makers in real time studies* was determining the START treaty drawdown requirements. The START negotiating team knew where the superpowers were today and the end point, but they needed milestones to measure treaty compliance. So Doug Owens, Bob Bivins, and I built a linear programming model to evaluate the different alternative drawdown milestones. We showed one set of drawdown milestones was the most robust solution. Our recommendation became a part of the treaty. Of course the study was classified at the time. We later published the study when it was declassified (Owens, D., Parnell, G., and Bivins, R., "Strategic Arms Reduction Treaty (START) Drawdown Analyses," *Operations Research,* Vol 44, No. 3, May–June 1996, pp. 425–434).

BOB SHELDON: What was the size of that problem? How many variables and constraints did you work with? Hundreds or thousands?

GREG PARNELL: Hundreds. There were many different force structures, delivery vehicles, and weapons, so we had a large number of constraints. You had to have constraints for each milestone. Since we had all the databases and knowledgeable analysts, we did the study in a weekend.

BOB SHELDON: Was one of the guys proficient with an LP solver?

GREG PARNELL: Yes. We used LINDO.

BOB SHELDON: Who in the chain of command did you brief?

GREG PARNELL: Initially, we only briefed our two-star. Because it was so close hold within the negotiation chain, it went through the START negotiating team. We briefed at the 06 level, and he took it from there, using our charts to explain it to the State Department negotiators. One of the team, Col Jae Engelbrecht, became one of my good friends. We later worked together on Air Force 2025 and with Toffler Associates.

BOB SHELDON: What was the impact of the end of the cold war on AFSAA?

GREG PARNELL: I led an interesting study for Maj Gen Harrison. In early 1990, it became increasing clear that we could not base our force structure analysis on potential NATO/Warsaw Pact conflicts in the Fulda Gap and Soviet/U.S. strategic nuclear balances. We looked for a new framework for force structure analysis. We came up with a force quality methodology (similar to today's capability based planning). We focused on measuring the force qualities we would need in the future for a variety of alternative scenarios. Unfortunately, our briefing had a long convoluted title.

Maj Gen Harrison liked the content but was not comfortable with the title. Later that day, after reading the new Air Force white paper: *Global Reach, Global Power,* I went jogging. While jogging, I figured out what to do. I returned to the office and changed the title of our briefing to *Analyzing Global Reach, Global Power* (Parnell, G. and Eilers, R., "A Methodology for Analyzing Global Reach—Global Power," White Paper, AF Center for Studies & Analyses," Air Force Studies and Analysis Agency, 1990).

I immediately took the new briefing into Maj Gen Harrison. He looked at the new title and said, "That's great." He took the briefing to the Assistant Vice, who said, "Great." The A/Vice took it to the Vice Chief of Staff and in same day it was in the Chief of Staff's package for overnight reading.

We ended up giving the briefing to most of the Air Force leaders in the Pentagon. We had one hour with General Larry Welch, who was the Chief. I briefed for half an hour and General Welch talked to us for half an hour about what he wanted us to do. We incorporated his ideas. Then General Dugan took over, we took the briefing to him. We spent an hour with him and he sent us to brief Secretary of the Air Force Don Rice. I went to schedule it on Secretary Rice's calendar. Rice happened to walk out of his office and his aide asked the Secretary about the briefing. Secretary Rice looked at the title and told the aide to schedule **four hours** for the briefing.

He spent several hours with us. This was an important lesson for me. Secretary Rice wanted to spend quality time with us because he wanted to get his analytical organization focused on his vision. He heard our ideas and gave us clear guidance on what he expected us to do.

It took AFSAA a year to analytically refocus and change our databases with new scenario data. Greg McIntyre led that important effort.

BOB SHELDON: You say Dr. Rice gave you directions in that four-hour session after you briefed him. What kind of directions did he give you?

GREG PARNELL: Both Welch and Rice wanted us to focus on the force quality issues. They believed that quantities would be determined more by budget than before. Since development programs take so long, it is critical to get the right force quality in development. Good examples of force qualities were stealth, continuous intelligence, precise navigation, and precision weapons. Of course, there are synergies, knowing precise target locations, using GPS, and dropping dumb bombs has been very effective.

BOB SHELDON: Let me backtrack. When did you first go to a MORS Symposium?

GREG PARNELL: I went to a MORS Symposium in about 1986 and got involved in working groups during my AFIT assignment. Clay Thomas and Jim Bexfield got me into MORS leadership. Clay was the Chief Scientist and Jim Bexfield was Chief Analyst of the Air Force Studies and Analyses Agency while I was at AFIT. They visited us every year to bring thesis topics. They became life-long mentors, as I'm sure Clay was to you. While I was at ICAF, they helped me get elected to the MORS Board of Directors. I became a director in 1989 as I started at Studies and Analyses.

BOB SHELDON: You've done a number of tutorials at MORS. When did you start doing those?

GREG PARNELL: I started in about 1996. The tutorials were about Value-Focused Thinking using multiple objective decision analysis.

BOB SHELDON: What was your initial impression of MORS when you first attended a MORS Symposium?

GREG PARNELL: I was very impressed from the start by the MORS staff (Dick Wiles, Natalie, and Cynthia) and the quality of the participants. I thought it was just a great professional society; much better organized and more customer focused than other professional societies.

BOB SHELDON: Do you recall some of your early MORS committees?

GREG PARNELL: I was on the Prize Committee the first year. Then I went through the meetings side. I was the Working Group/Composite Group Chair my second year. I met a lot of MORS leaders coordinating all the working groups and composite groups. My next job was Vice President for Meeting Operations. I led a major working group realignment to provide better post-cold war focus. The next year I served as President, then Past President. I learned that the best job in MORS is Past-Past President! The job has good prestige and no assignments!

BOB SHELDON: As VP for Meeting Operations, did you face any hurdles?

GREG PARNELL: Things were organized pretty well. I inherited a good slate of meetings. We generated more ideas and we executed the ones the sponsors wanted.

BOB SHELDON: I think you were the second active duty officer to become President of MORS. Did you have any trepidation about conflict of interest for active duty officers?

GREG PARNELL: I can't recall any concerns or any conflict of interest issues that occurred during my tenure.

BOB SHELDON: Any other tough issues during your time on the MORS Executive Council?

GREG PARNELL: I was fortunate to follow E. B. Vandiver and Van had a great plan to build on. My theme for the year was quality support to our customers. I tried to continue to focus our activities on our clients and analysts. We also moved the *Military Operations Research* Journal closer to realization.

BOB SHELDON: You briefed each of the sponsors as the MORS President? Remember any specific feedback you got from them?

GREG PARNELL: I do remember that they gave us clear guidance on the meetings they wanted. I do not recall any other specific feedback.

BOB SHELDON: You were involved in developing a workshop for Admiral Owens. How did MORS get such a quick turnaround for the meeting?

GREG PARNELL: I think it was the year after I was President. When you have the Vice Chairman of the Joint Chiefs ask you to help, everybody was motivated. People thought it was a great opportunity to participate.

BOB SHELDON: Do you recall any of the fallout after that meeting?

GREG PARNELL: Change in the military (or any large organization) is difficult. Some did not understand Admiral Owens' joint warfare objectives and some opposed these objectives actively or passively. Some wanted to continue to think about major service systems and not have to show how service systems supported joint warfare.

BOB SHELDON: Let's get back to your time at AFSAA. Talk about the transition when Brig Gen Eberhardt took over and AFSAA became part of PE. Was it difficult to make that transition?

GREG PARNELL: Very difficult. Maj Gen George Harrison, AFSAA commander, put together a team which I led. Dan Barker was a key member of the team. He gave us one day to reorganize Studies and Analyses. We had about ten people from all the AFSAA divisions on the team. There were a couple guidelines. First, we had to have an organizational element providing resource allocation analysis support to PE. Second, General McPeak, then Chief, had a new rule we had to follow—colonels could not work for more than one colonel in their management chain. This was a big deal. In Studies and Analyses we had Colonels as directors, deputy directors, and division chiefs. Since the Commander of AFSAA would be a colonel we could have only divisions. (Later Tom Cardwell became the Commander of AFSAA.)

I'll never forget the expression on Maj Gen Harrison's face when we showed him the number of analysts he had out of 150 people. We were able to reorganize to about 100 people without a significant change in the number of analysts. However, we since developed about a 30 person Resource Analysis Division that reduced the number of analysts available for previous AFSAA missions. Our reorganization plan was briefed to Maj Gen Harrison and Brig Gen Eberhardt. The plan was accepted without change.

BOB SHELDON: You did the planning for the transition of AFSAA under PE. How was the actual transition?

GREG PARNELL: The transition was difficult. Working for PE changed the culture of the organization. The culture of Studies and Analyses was about the number of stars that you could brief the results of your study. We now had a BG, and not MG, who was focused on resource allocation to develop the Air Force Program. His focus was the Planning Programming Budgeting System process, not briefing studies. So the people in the new Resource Analysis Division were getting all the visibility with PE. During the major programming phase, PE focused only on resource allocation. After these intense phases, the PE leadership took leave and then started preparing for the next cycle.

My division's job was to provide resource allocation analyses to support senior leader decision-making. To make a large number of program decisions (50–100 per briefing), we had to boil everything down to one analysis chart. AFSAA analysts would do a briefing and then one of my guys would work with them to boil it down to one chart. If an explanation was required, I would explain the chart since the analysis team was not in the briefing. This was a major culture change. The AFSAA analyst may have had a direct impact on a budget decision but did not get to brief his/her 30 charts to several generals.

BOB SHELDON: I understand that Brig Gen Eberhardt used different ways of introducing you to the Secretary of the Air Force as compared to the Chief of Staff of the Air Force. Explain that.

GREG PARNELL: At that time, General McPeak was the Chief and Don Rice was the Secretary. General McPeak did not like the word "analysis." On the other hand, he liked good data to help him make decisions. So when I would brief General McPeak, Brig Gen Eberhardt would say, "This is Greg Parnell from Air Force Programs and Evaluation." When we took the same briefing to the Secretary of the Air Force, Don Rice, former President of RAND, he would say, "This is Greg Parnell from Air Force Studies and Analyses."

BOB SHELDON: Give us more of your impressions about Dr. Rice.

GREG PARNELL: Don Rice was a great Secretary of the Air Force. He led the Air Force in a very critical period. He was easy to brief and asked great questions. He was very involved in resource allocation. He wanted to get programs in the right mission bins and then look at tradeoffs within the missions. Most of our analysis for him was within the missions, e.g., space systems, conventional forces, nuclear forces. He and the Chief decided the relative allocation between missions.

I remember one amusing story. We briefed the Air Force budget to him with the Vice Chief and all the three-stars. This briefing was very long. You could tell people were starting to squirm in their seats. Finally, his aide said, "Sir, there's a telephone call you need to take." The Air Force senior leaders literally knocked chairs over as they rushed to the bathrooms!

BOB SHELDON: You spent a number of years in the nuclear analysis. Did you ever get any advice from Lt Gen Glenn Kent?

GREG PARNELL: Yes. Two of my early mentors Colonels Bill Crabtree and Carl Case connected me with Glenn Kent. When I was managing the MX Transporter I had built a chart to explain my development program. When the chart was presented to the Air Force Scientific Advisory Board, Lt Gen Kent said, "That was a great chart, who developed it?" And they said, "This young guy out of ballistic missile office." Subsequently, Colonels Crabtree and Case arranged for me to spend one full day with Glenn Kent. He provided some lessons that I still use:

- "There are benefits and there are costs. Never mix the two."
- "Always look at the maximum benefit for fixed cost or the minimum cost for fixed benefit."
- "If a briefer does not precisely define the terms used in the briefing, it will be a waste of time to hear their briefing."

When I was department head at AFIT, every year we invited him to talk to students and faculty. It was always a great visit. Here is one of my favorite Glenn Kent stories. We were having dinner at the Officers' Club at Wright Patt. At the time, he was strongly advocating his 'Strategy to Task' methodology. He said "Greg, what is your opinion of Strategy to Task." So I thought carefully and said "I like it because it's a very logical process starting with the strategy through the operations concept to the tasks. However, magic has to occur in about two places." He looked at me, and without missing a beat, said, "Only two?"

BOB SHELDON: At the end of your tour at AFSAA, did you want to return to AFIT or did somebody twist your arm?

GREG PARNELL: No, I wanted to go to AFIT. Maj Gen Eberhardt wanted me go to J-8 with him but finally agreed to let me go. After the Air Force, I planned to be a professor and a consultant. The best transition path for me was to go back to AFIT as department head.

BOB SHELDON: I take it you really enjoyed your first tour teaching at AFIT? What was it you found so attractive about the teaching environment?

GREG PARNELL: Yes, I did. I have always enjoyed teaching and mentoring students. I liked the variety of problems and the ability to choose the problems I work on. I usually did not get to choose my problems in Studies and Analyses. I also liked the flexible schedule. I could take off at 4:00 to coach my son's soccer team, spend time with the family, and then prepare my lecture at 9:00 at night (usually for the next day!).

BOB SHELDON: One of the ongoing issues is keeping AFIT responsive to the Air Force's needs. How did you address that issue?

GREG PARNELL: I designated a faculty member as liaison with every major command analysis office. Their job was to coordinate thesis topics, slots for graduates, and trips to AFIT. I tried to align the assignments with research interests. We also encouraged students to do funded research for the major commands for their thesis work.

BOB SHELDON: Can you think of any specific things you did to respond to Air Force needs during your tenure at AFIT?

GREG PARNELL: Yes, that is another good story. Remember, I mentioned Jae Engelbreht from our arms control studies. Shortly after I had gotten to AFIT, Jae called me up on a Thursday afternoon. He said, "Greg, I need your help. We have done this study called SPACECAST 2020. We are 11 months into a 12-month study and we have no idea how to evaluate the alternatives. Our study director, Lt Gen Jay Kelley has to brief General McPeak, who commissioned the study. Can we fly out tomorrow morning and meet with you Friday afternoon and Saturday?" My friends call this story "Desperate men come to Dayton, Ohio."

The study alternatives were space system concepts. Engineering data did not exist to evaluate the alternatives using models and simulations. We developed a multiple objective decision analysis methodology to rank the alternatives. Roger Burk, one of my faculty and now a colleague at USMA, worked with me on the study (Burk, R. C. and Parnell, G. S., "Evaluating Future Space Systems and Technologies," *Interfaces,* Vol 27, No 3, May–June 1997, pp. 60–73).

The next week Roger and I went to Maxwell to brief Lt Gen Jay Kelley and all his senior staff. I knew it was important when he said, "This discussion has no time limit." After a detailed briefing and lots of discussion, he approved our plan. Roger worked full time for a month on the analysis. The study and the analysis were very successful.

This study led to a major AFIT role in Air Force 2025 which I supported as a consultant to the Air Force (Jackson, J. A., Parnell, G. S., Jones, B. L., Lehmkuhl, L. J., Conley, H., and Andrew, J., "Air Force 2025 Operational Analysis," *Military Operations Research,* 1997, Vol 3, No 4, pp. 5–21). AFIT worked for a full year on the second study. Air Force 2025 made an important impact on the Air Force.

BOB SHELDON: After you had been at AFIT, you decided to hang up your hat and retire. What was your decision algorithm for deciding what to do at that point?

GREG PARNELL: That's a great question. I took up golf while I was in Dayton because my sons wanted to play golf. My good friend Bill Rowell got the three of us into golf. We decided we wanted to move to a more golf friendly climate. Timing for my second son's high school was also a consideration. I decided to retire at 25 years to give him three years at his new high school. I interviewed with a several schools but decided to go Virginia Commonwealth University (VCU) in Richmond, VA.

BOB SHELDON: Was it traumatic to retire from the Air Force, or was it something you looked forward to?

GREG PARNELL: It was not traumatic. We retired near Fort Lee and a lot of our friends were retired officers. I also started doing consulting on military problems.

BOB SHELDON: What courses did you teach at Virginia Commonwealth?

GREG PARNELL: I taught two existing courses (deterministic and stochastic methods) and I added two courses. The first was decision and risk analysis (similar to AFIT course but added risk analysis material). I also developed a new multiple objective decision analysis course. This is the course I mentioned earlier that Jack Jackson introduced at AFIT.

BOB SHELDON: You've been teaching Value-Focused Thinking (VFT) to your students and in professional short courses. When did you first pick that up?

GREG PARNELL: Multiple objective decision analysis (Keeney, R.L. and Raiffa H. *Decision Making with Multiple Objectives: Preferences and Value Tradeoffs*, New York: Wiley, 1976) is the mathematics behind Value-Focused Thinking (Keeney, R.L. *Value-Focused Thinking: A Path to Creative Decisionmaking,* Cambridge, Massachusetts: Harvard University Press, 1992). We use VFT to define our values and use our values to create better alternatives. SPACECAST 2020 used multiple objective decision analysis to evaluate the alternatives but in one month, we did not have time to use VFT to improve the alternatives. Air Force 2025 was my first use of VFT. While at VCU, I started teaching multiple objective decision analysis short courses. In the last 10 years, I have taught over 30 of the courses for government and industry.

BOB SHELDON: Did you ever work with Ralph Keeney, developer of VFT?

GREG PARNELL: I know Ralph well from the Decision Analysis Society and journal editorial activities.

BOB SHELDON: How did you like the teaching at VCU as opposed to AFIT?

GREG PARNELL: I enjoyed teaching at VCU. We had a small operations research program with many of our graduate students being part time with jobs in the Richmond area. Part time students were not able to focus on research as much as the full time AFIT students.

BOB SHELDON: So after four years, did you jump at the chance to go to West Point?

GREG PARNELL: No. When the first West Point faculty member asked me to apply, I said no. VCU had treated me well and my wife enjoyed Richmond. Then a second faculty member called. Again, I said no. I had just gotten tenure and an academic promotion. Finally, the department head, COL Jim Kays, called. After talking to Jim, I went home and told my wife that maybe the Lord was trying to tell us something. So we talked about it and I took Eileen with me to the interview. When I was offered the job, we decided to go to West Point.

BOB SHELDON: Have there been a lot of changes in the Systems Engineering Department since you have been here?

GREG PARNELL: Yes. The department was started in 1989 by Jim Kays. The second department head, Mike McGinnis, arrived the same summer as I did. It has been my honor to serve with Mike. He and his leadership team built on a solid foundation and developed many new programs to take the department to the next level (McGinnis, Michael L., Ph.D., "Transforming the Department: 1999–2004" Technical Report No. DSE-TR-04-29, DTIC #: ADA424113, Operations Research Center of Excellence, Department of Systems Engineering, West Point, NY, May 2004).

BOB SHELDON: Tell us about the Chair that you filled.

GREG PARNELL: I have just finished a six-year tour as the Class of 1950 Chair of Advanced Technology in the Department of Systems Engineering. The class donated funds to the academy to fund a chair. The chair holder would rotate between engineering departments. I was selected as the first chair. It was a great opportunity to meet and work with the Class of 1950. In summer 2005, General Paul Kern, USA, retired, became the second chair holder. GEN Kern will teach in the Civil and Mechanical Engineering Department.

BOB SHELDON: What's your position in the fall?

GREG PARNELL: I'll be on sabbatical for a year. Then I'll return as a Title 10 Professor.

BOB SHELDON: What's a Title 10 Professor?

GREG PARNELL: All of the civilian professors and key academic staff are Title 10 instead of GS. We serve on six-year renewable contracts with a separate pay scale.

BOB SHELDON: How would you compare teaching here at West Point to AFIT?

GREG PARNELL: It's undergraduate versus graduate material. Also, AFIT students were older, usually married, and more focused on academics. The cadets are very smart and (generally) hard working. Cadets have many other activities. However, they're great young men and women, and it's an honor to be here.

BOB SHELDON: What about the West Point area appeals to you?

GREG PARNELL: We live in a very nice village called Cornwall-on-Hudson just over the mountain north of West Point. We are 45 minutes from "The City." For seven months a year it is great. The Hudson Valley is relatively cool in summer, lovely in spring, and spectacular in the fall. Then there is winter.

BOB SHELDON: But you lived in Dayton so you're used to that.

GREG PARNELL: That's true.

BOB SHELDON: Tell us about your association with Toffler Associates.

GREG PARNELL: While in the Air Force, I worked on SPACECAST 2020 for Lt Gen Jay Kelley, Air University Commander. Dick Szfranski, and Jae Engelbrecht also had key roles in the study. After I retired, I worked on Air Force 2025 with the same team. The next year all three retired and helped found Toffler Associates, a new consulting firm founded by Alvin and Heidi Toffler, the well known futurists and authors of *Future Shock* and the *The Third Wave,* and Tom Johnson, a very experienced consultant.

After retiring I had consulted with IDA and TASC. Then Dick Szfranski asked me to consult with Toffler Associates. I decided it would be fun to work with them and a great opportunity to meet and work with Alvin and Heidi. I worked with Toffler Associates on a variety of strategic planning and some decision analysis projects for six years.

BOB SHELDON: How did you decide to change to Innovative Decision Inc.?

GREG PARNELL: In late 2003, Terry Bresnick, President of Innovative Decision, Inc., approached me about joining a new decision analysis consulting firm that he and other partners were forming. One partner, Joe Tatman, was a good friend. In addition, I had known Terry and Dennis Buede for years. I had been thinking about more technical work and this was an ideal opportunity. Since then, IDI has grown to about 20 people.

BOB SHELDON: What customers do you support?

GREG PARNELL: IDI works with intelligence, defense, homeland security, and commercial companies. In addition, they teach decision analysis and systems engineering courses. I work part-time on IDI projects. I do some travel and do other work at home.

BOB SHELDON: What courses are you teaching now?

GREG PARNELL: I teach decision analysis and operations research courses. We have no more than 18 students per section. I teach two sections per semester and lead a senior cadet capstone research project with three to five cadets per year.

BOB SHELDON: What kind of capstone projects have you led?

GREG PARNELL: I have done a variety of them in the six years. I started with an intelligence project for the Army Ground Intelligence Center. Then we worked on Army resource allocation projects for G-3 for three years. For the last two years, I have worked on base realignment and closure implementation.

BOB SHELDON: What major studies have you been involved in at West Point?

GREG PARNELL: I have worked on three major studies. The first study was the Installation Management Study (IMA) led by then COL Tim Trainor. The purpose of the IMA study was to assess the regional organization structure of IMA. This was a six month study. The second study was the Residential Community Initiative (RCI) Study led by COL Bob Powell. The purpose of this six month study was to review the RCI program and make recommendations for improvement. Both of these studies were done for the ASA (Installations and Environment). The third major study I worked on was the Army Base Realignment and Closure 2005 study. I spent about three years working on the study. I supported Dr. Craig College, COL Bill Tarantino, and LTC Lee Ewing.

BOB SHELDON: You've also been involved with INFORMS. Tell us how you got involved in that.

GREG PARNELL: Like MORS, I got involved by going to the meetings. My INFORMS involvement was in three areas: the Military Applications Society (MAS), the Decision Analysis Society (DAS), and the Richmond-Tidewater Chapter of INFORMS. I usually participated in MAS and DAS sessions at INFORMS. I helped establish the Richmond-Tidewater chapter of INFORMS and served as President. Shortly after coming to West Point I got elected to the councils of both MAS and DAS. I chose to focus on the DAS because decision analysis is my primary research area. I was elected President of DAS in 2004. I will serve as President until October 2006. Then I will serve as Past President for two years. After being president of DAS and MORS, I plan to retire from professional society elected offices! Both have been great opportunities for service and building friendships.

BOB SHELDON: How large is the Decision Analysis Society?

GREG PARNELL: We are currently the second largest society in INFORMS with about 940 members.

BOB SHELDON: Do you hold your own separate meetings?

GREG PARNELL: We hold our meetings in conjunction with INFORMS. We also support decision analysis tracks at international meetings.

BOB SHELDON: Does DAS tend to be mostly academics?

GREG PARNELL: Professors and students make up 57% of the membership. The rest are practioners.

BOB SHELDON: Since you've been involved in several different national societies, talk about MORS as a professional society compared to the others.

GREG PARNELL: MORS is my favorite professional society. MORS is very well run and mission focused. I have made a lot of great friends through MORS. My contacts in MORS led directly to my job at West Point.

BOB SHELDON: You just received the Thomas Award, and Clay Thomas was one of your mentors. What was your relationship and what did you learn from him?

GREG PARNELL: Clay could always see the essence of the problem and would always provide sound advice. In AFSAA, when I started a new project, I would take my ideas to Clay for advice. I would sit in his office and just explain what I was doing and get his thoughts. He was always very helpful. Many times, he would say, "Well, we wrote a paper on that." He would shuffle through a pile of papers and pull out the paper! Also, he always gave sage advice on MORS activities.

BOB SHELDON: Now you mentor young analysts. Do you have any young analysts that you've mentored that have done well and you would like to claim you have influenced?

GREG PARNELL: I enjoy working with and mentoring young analysts. I like to claim I influenced all of them! I know I learn a lot from each of them. Jack Jackson was a student of mine as a young major. He worked for me at AFIT and we worked on research projects after I retired. He currently works for SAIC. We sponsor his son Daniel at West Point. I hired Jack Kloeber as a new Army LTC at AFIT. We have worked together on several studies and professional society actives. He's currently in charge of pharmaceutical R&D portfolio analysis for Johnson & Johnson. More recently, I worked closely with several great young officers at West Point. LTC Barry Ezell, one of these outstanding officers, is now at Army Capabilities and Integration Center. Another officer, MAJ Brian Stokes now works for Army G-8. On BRAC, I worked closely with LTC Lee Ewing, now a professor at NPS. Lee did a great job on BRAC. At IDI, I work closely with Don Buckshaw and Bob Liebe, two outstanding decision analysts.

BOB SHELDON: Since MORS helps foster mentoring, any advice about how we can do a better job of that?

GREG PARNELL: I like the junior/senior analysts sessions that we have. I think those are very useful. But most mentoring happens informally. For example, Jim Bexfield mentored me and helped me get elected to the MORS Board. Later, he asked me to be a consultant for him when he was at IDA.

BOB SHELDON: You're presenting a paper this week. Will you be sitting in some of the other working group sessions?

GREG PARNELL: I will be attending the Decision Analysis Working Group.

BOB SHELDON: Talk about your duties as the *MOR* Journal editor.

GREG PARNELL: Our first editor, Peter Purdue, gets the primary credit for establishing the *MOR* Journal. Jim Kays and E.B. Vandiver played very important roles. I was the second editor. My job was to maintain quality and increase production. We had to obtain more quality papers and get more reviewers involved to speed up the review process. I viewed it was my job to talk people into writing papers and to select great associate editors. I served as Editor for over five years until the selection of Dick Deckro. Dick has done a great job with the journal.

BOB SHELDON: Did you just use the connections you had from your MORS years, or did you find some new contacts?

GREG PARNELL: Both. We took award papers and published them each year. I used every opportunity to encourage people to write up good studies for publication. Each year, I deputized every Board member to help. I told them you either have to find a paper or write a paper! It was a lot of work but we got the production rate up thanks to the great work of the authors and the associate editors.

BOB SHELDON: We always have a tradeoff between good peer review of the papers and timely turnaround. How did you manage the tradeoff between those two?

GREG PARNELL: It's all about people. I found good people that could do quality reviews in a timely manner. To ensure quality each paper had to be recommended for acceptance by two associate editors: one from the operations research technique and one from the problem domain. For example, a paper using an optimization model for a naval application would have two reviewers: an optimizer and a naval expert. The optimizer's job was to assess the quality of optimization work. The naval expert's job was to assess the value of the work to the naval application area.

BOB SHELDON: How many of them came back with two yes's?

GREG PARNELL: We had an acceptance rate of about 45%. Almost no papers were accepted on the first review. Papers were usually accepted subject to revisions.

BOB SHELDON: How do you view our *MOR* technical papers as compared to technical papers in other journals?

GREG PARNELL: I think the journal is very good. Our niche is military operations research applications. We also publish some articles on military OR theory and heritage. Although we are a relatively new journal by academic standards, the journal has established a record of quality articles. I believe it is the best journal for military OR applications in the world.

BOB SHELDON: What other professional service activities have you been involved in?

GREG PARNELL: Since 2002, I have been a member of the Technology Panel of the National Security Agency Advisory Board. I spend two days a month supporting them.

BOB SHELDON: Any other parting shots?

GREG PARNELL: As I reflect back on my career, I have been very fortunate. First, I have had wonderful support from my wife and family. They have let me pursue education opportunities and take new job challenges. Second, I was able to have operational and management experience <u>before</u> I became an operations research analyst and professor. This gave both: experience and credibility. Third, I was very fortunate to have mentors that gave me great advice along the way. Fourth, I have had the opportunity to work on very important projects with great people. I have worked on Air Force, Army, intelligence, environmental, and homeland security problems. I have made life long friendships. Fifth, I have had significant opportunities for professional service in professional societies and advisory boards. These have also led to wonderful friendships. Finally, I have had many opportunities to mentor young officers and civilians.

About our Authors

PROBABILITY MODELING OF AUTONOMOUS UNMANNED COMBAT AERIAL VEHICLES (UCAVs)

by Moshe Kress, Arne Baggesen and Eylam Gofer

Moshe Kress teaches advanced combat modeling and probability models at the NPS. His research interests are combat modelling, homeland security problems, biodefense and logistics. He is the Military and Homeland Security Area Editor for *Operations Research* and an associate editor for NRL and *MOR*. You can learn more about him by visiting his web site http://www.nps.navy.mil/orfacpag/resumePages/Kress.htm.

Arne Baggesen is an active duty Lieutenant Commander in the German Navy. He is working for the Sensors and Weapons Department of the German Naval Office. Among his responsibilities are naval contributions to Ballistic Missile Defense, review of sensor suites for new ship-designs, management support of in-service radar systems, and naval systems-simulations. His academic background is a Practical Engineer degree in mechanical engineering from the "Helmut Schmidt University", Hamburg, Germany, and a MSc in "Modelling Virtual Environments and Simulations" from the Naval Postgraduate School Monterey, California.

Eylam Gofer is an analyst at the center for military analyses (CEMA) in Israel. He is currently the head of the land warfare and intelligence branch. His current research interests are decision making under uncertainty, combat modeling and network congestion games.

DISTORTED RISK MEASURES WITH APPLICATION TO MILITARY CAPABILITY SHORTFALLS

by Edwin J. Offut, Jeffrey P. Kharoufeh and Richard F. Deckro

Edwin J. Offutt is a Major in the U.S. Air Force and is currently Chief of the F-15E Strike Eagle Team at Wright Patterson AFB, Ohio. Prior to his current assignment, Ed earned an M.S. in Operations Research at the Air Force Institute of Technology.

Dr. Jeffrey P. Kharoufeh is an Associate Professor of Operations Research at the Air Force Institute of Technology where he teaches stochastic modeling and analysis as well as queuing theory. He holds a doctorate in Industrial Engineering and Operations Research from Penn State University. Dr. Kharoufeh's primary area of research interest is the modeling, analysis and control of stochastic systems. He currently serves as an Associate Editor for *Operations Research Letters* and the *IEEE Transactions on Reliability.*

Dr. Richard F. Deckro is a Professor of Operations Research in the Department of Operational Sciences at the Air Force Institute of Technology. He holds a BSIE from the State University of New York at Buffalo, and an MBA and a Doctorate of Business Administration in Decision Sciences from Kent State University. Dick's research, teaching and consulting interests are in the areas of information operations and information assurance, reconstruction and stabilization, measures of effectiveness and assessment, behavioral modeling, social networks, applied mathematical programming and optimization, project and program management, campaign modeling, technology selection and management, scheduling, network models, advanced manufacturing methods, multi-criteria decision making, and decision analysis.

Dick is the Area Editor for Services for *Computers & Industrial Engineering* and is currently the President of the Military Applications Society (MAS) of INFORMS. It is also Dick's pleasure and honor to serve as the Editor of *Military Operations Research.*

ESTIMATING TOTAL PROGRAM COST OF A LONG-TERM, HIGH-TECHNOLOGY, HIGH-RISK PROJECT WITH TASK DURATIONS AND COSTS THAT MAY INCREASE OVER TIME

by Gerald G. Brown, Roger T. Grose, and Robert A. Koyak

Gerald G. Brown is Distinguished Professor of Operations Research at the Naval

Postgraduate School, where he has had the same job and phone number since 1973. He is an optimizer.

Roger T. Grose is a Major in the Australian Army. He recently completed a tour as the ORSA supporting Australian troops in the Middle East. In addition to his Masters degree in Operations Research from the Naval Postgraduate School, he likes to collect degrees from Australian universities.

Robert A. Koyak is Associate Professor of Operations Research at the Naval Postgraduate School. When he is not running he tries to fit in teaching and research as a statistician.

United States Postal Service Statement of Ownership, Management and Circulation (PS Form 3526)

1. Publication Title: Military Operations Research
2. Publication Number: 0275-5823
3. Filing Date: 30 September 2006
4. Issue Frequency: Quarterly
5. Number of Issues Published Annually:
6. Annual Subscription Price: Domestic-$60.00/year and Foreign-$120.00/year
7. Complete Mailing Address of Known Office of Publication: Military Operations Research Society, 1703 N. Beauregard Street, Suite 450, Alexandria, VA 22311-1745
8. Complete Mailing Address of Headquarters or General Business Office of Publisher: Military Operations Research Society, 1703 N. Beauregard Street, Suite 450, Alexandria, VA 22311-1745
9. Full Names and Complete Mailing Addresses of Publisher, Editor, and Managing Editor
Publisher: Military Operations Research Society, 1703 N. Beauregard Street, Suite 450, Alexandria, VA 22311-1745
Editor and Managing Editor: Dr Richard Deckro, Military Operations Research Society, 1703 N. Beauregard Street, Suite 450, Alexandria, VA 22311-1745
10. Owner: Military Operations Research Society, 1703 N. Beauregard Street, Suite 450, Alexandria, VA 22311-1745 and Institute for Operations Research and the Management Sciences, 7240 Parkway Drive, Suite 310, Hanover, MD 21076
11.Known Bondholders, Mortgagees, and Other Security Holders Owning or Holding 1 Percent or More of Total Amount of Bonds, Mortgages, or Other Securities. None
12. Tax Status (For completion by nonprofit organizations authorized to mail at nonprofit rates). The purpose, function, and nonprofit status of this organization and the exempt status for federal income tax purposes: Has Not Changed During Preceding 12 Months
13. Publication Title: Military Operations Research
14.Issue Date for Circulation Data Below: Volume 11 Number 2

15. Extent and Nature of Circulation	*Average No. Copies Each Issue During Preceding 12 Months*	*No. Copies of Single Issue Published Nearest to Filing Date*
a. Total Number of Copies	3,939	3,445
b. Paid and/or Requested Circulation		
(1) Paid/Requested Outside-County Mail Subscriptions Stated on Form 3541. (Include advertiser's proof and exchange copies)	3,632	3,280
(2) Paid In-County Subscriptions Stated on Form 3541(Include advertiser's proof and exchange copies)	0	0
(3) Sales Through Dealers and Carriers, Street Vendors, Counter Sales, and Other Non-USPS Paid Distribution	2	0
(4) Other Classes Mailed Through the USPS	32	6
c. Total Paid and/or Requested Circulation	3,666	3,286
d. Free Distribution by Mail Samples, complimentary, and other free)		
(1) Outside-County as Stated on Form 3541	0	0
(2) In-County as Stated on Form 3541	0	0
(3) Other Classes Mailed Through the USPS	60	30
e. Free Distribution Outside the Mail (Carriers or other means)	100	40
f. Total Free Distribution	160	70
g. Total Distribution	3,826	3,356
h. Copies not Distributed	113	89
i. Total	3,939	3,445
j. Percent Paid and/or Requested Circulation	96%	98%

16. This statement of Ownership will be printed in Volume 11 Number 4 *of this publication.*

Military Operations Research

A publication of the Military Operations Research Society

The Military Operations Research Society is a professional society incorporated under the laws of Virginia. The Society conducts a classified symposium and several other meetings annually. It publishes proceedings, monographs, a quarterly bulletin, *PHALANX*, and a quarterly journal, *Military Operations Research*, for professional exchange and peer criticism among students, theoreticians, practitioners and users of military operations research. The Society does not make or advocate official policy nor does it attempt to influence the formation of policy. Matters discussed or statements made in the course of MORS symposia or printed in its publications represent the opinions of the authors and not the Society.

Editor
Dr. Richard F. Deckro
Department of Operational Sciences
Air Force Institute of Technology
AFIT/ENS, Bldg 641
2950 Hobson Way
Wright-Patterson AFB, OH 45433-7765

Publisher
Corrina Ross-Witkowski, Communications Manager
Military Operations Research Society
1703 N. Beauregard Street, Suite 450
Alexandria, VA 22311

Editors Emeritus
Dr. Peter Purdue
Naval Postgraduate School

Dr. Gregory S. Parnell, FS
United States Military Academy

Associate Editors
Dr. Patrick D. Allen
  General Dynamics Advanced Information Systems
Dr. Paul F. Auclair
  LexisNexis
Dr. Stephen J. Balut
  Institute for Defense Analyses
Dr. Jerome Bracken
  IDA, RAND & Department of State
Dr. Alfred G. Brandstein
  The MITRE Corporation
Dr. Gerald G. Brown
  Naval Postgraduate School
Dr. Marion R. Bryson, FS
  North Tree Fire International
Maj Stephen P. Chambal
  AFRL
Dr. Charles A. Correia
  Virginia Commonwealth University
Dr. Paul K. Davis
  RAND and Pardee RAND Graduate School
Dr. Jerry Diaz
  Consultant
Prof. Patrick J. Driscoll
  United States Military Academy
LTC Barry C. Ezell, Ph.D.
  Army Capabilities Integration Center
Dr. Bruce W. Fowler
  U. S. Army Research and Development Command
Dr. Mark A. Gallagher
  United States Strategic Command
Dr. Dean S. Hartley, III
  Hartley Consulting
Dr. Raymond R. Hill, Jr.
  Wright State University
Prof. Wayne P. Hughes, Jr., FS
  Naval Postgraduate School
Dr. Jack A. Jackson
  Science Applications International Corporation
Dr. James L. Kays
  Naval Postgraduate School
Dr. Moshe Kress
  Center for Military Analyses, Israel & Naval Postgraduate School
Dr. Andrew G. Loerch
  George Mason University
Dr. James K. Lowe
  US Air Force Academy
Mr. Brian R. McEnany, FS
  Science Applications International Corporation
Dr. Michael L. McGinnis
  VMASC
Dr. James T. Moore
  Air Force Institute of Technology
Prof. W. Charles Mylander III
  United States Naval Academy
Dr. Kevin Ng
  Department of National Defence, Canada
Dr. Daniel A. Nussbaum
  Naval Postgraduate School
Dr. Edward A. Pohl
  University of Arkansas
Dr. Kevin J. Saeger
  Los Alamos National Laboratory
Dr. Robert S. Sheldon, FS
  Group W Inc.
Dr. Joseph A. Tatman
  Innovative Decisions, Inc.
Mr. Eugene P. Visco, FS
  Visco Consulting
Dr. Keith Womer
  University of Missouri – St. Louis

Military Operations Research, the journal of the Military Operations Research Society (ISSN 0275-5823) is published quarterly by the Military Operations Research Society, 1703 N. Beauregard Street, Suite 450, Alexandria, VA 22311-1745. The domestic subscription price is for one year and for two years; international rates are for one year and for two years. Periodicals Postage Paid at Alexandria, VA. and additional mailing offices.

POSTMASTER: Send address changes to *Military Operations Research*, the journal of the Military Operations Research Society, 1703 N. Beauregard St, Suite 450, Alexandria, VA 22311. Please allow 4–6 weeks for address change activation.

Military Operations Research is indexed in the following Thomson ISI® services: Science Citation Index Expanded (SciSearch®); ISI Alerting Services®; CompuMath Citation Index®.

© Copyright 2006, Military Operations Research Society, 1703 N. Beauregard St, Suite 450, Alexandria, VA 22311. ISSN Number 0275-5823

Printed on Recycled Paper

MILITARY OPERATIONS RESEARCH SOCIETY BOARD OF DIRECTORS

OFFICERS

President
Patrick J. McKenna*
USSTRATCOM/J821
mckennap@stratcom.mil

President Elect
John F. Keane*
Johns Hopkins University/APL
jack.keane@jhuapl.edu

Vice President Finance and Management (F&M)
Robert C. Holcomb*
Institute for Defense Analyses
rholcomb@ida.org

Vice President Meeting Operations (MO)
Gregory A. Keethler*
Lockheed Martin Missile and Fire Control
gregory.keethler@lmco.com

Vice President Professional Affairs (PA)
LTC Michael J. Kwinn*
United States Military Academy
michael.kwinn@usma.edu

Secretary
Dr. Niki C. (Deliman) Goerger
United States Military Academy
nicolette.goerger@usma.edu

Immediate Past President
Col Suzanne M. Beers*
AFOTEC Det 4/CC
suzanne.beers@afotec.af.mil

Executive Vice President
Brian D. Engler*
Military Operations Research Society
brian@mors.org

Vice President Administration
Natalie Strawn Kelly
Military Operations Research Society
natalie@mors.org

*** - Denotes Executive Council Member**

OTHER DIRECTORS

Dr. Patrick D. Allen, General Dynamics Advanced Information Systems
Lt Col Andrew P. Armacost, United States Air Force Academy
Dr. Theodore Bennett, Jr., Naval Oceanographic Office
Joseph C. Bonnet, The Joint Staff, J7/JTCD
Mary T. Bonnet, AD, Innovative Management Concepts, Inc.
Dr. W. Forrest Crain, HQDA, ODCS G-3/5/7, DAMO-CI
William H. Dunn, AD, Alion Science and Technology
Helaine G. Elderkin, FS, AD, Computer Sciences Corporation
Dr. Karsten Engelmann, Center for Army Analysis
John R. Ferguson, AD, SAIC
Michael W. Garrambone, General Dynamics
Debra R. Hall, General Dynamics
Timothy W. Hope, Alion Science and Technology
Jane Gatewood Krolewski, USAMSAA
William M. Kroshl, Johns Hopkins University/APL
Trena Covington Lilly, Johns Hopkins University/APL
Dr. Daniel T. Maxwell, Innovative Decisions
Dr. Willie J. McFadden II, AD, Booz Allen Hamilton
Lana E. McGlynn, AD, Consultant
Dr. Gregory A. McIntyre, Applied Research Associates, Inc.
Terrence McKearney, The Ranger Group
Kirk A. Michealson, Lockheed Martin
Anne M. Patenaude, DUSD(R)/RTPP
Dr. Steven E. Pilnick, Naval Postgraduate School
Mark D. Reid, MITRE
Cortez D. Stephens, MCCDC
Col Peter M. Vanden Bosch
Eugene P. Visco, FS, AD, Consultant
Corinne C. Wallshein, HQ USAF/A9
COL Richard I. Wiles, USA-ret, FS, AD

MILITARY OPERATIONS RESEARCH SOCIETY
1703 N. BEAUREGARD STREET
SUITE 450
ALEXANDRIA, VA 22311
Telephone: (703) 933-9070